Unit 3

:Basic Definition and Terminology, Set-theoretic operations, Fuzzy Sets, Operations on Fuzzy Sets, Fuzzy Relations, Membership Functions, Fuzzy Rules & Fuzzy Reasoning, Fuzzy Inference Systems, Fuzzy Expert Systems, Fuzzy Decision Making; Neuro fuzzy modeling- Adaptive Neuro-Fuzzy Inference Systems, Coactive Neuro-Fuzzy Modeling, Classification and Regression Trees, Data Clustering Algorithms, Rule base Structure Identification.

LECTURE-1

Fuzzy Sets Basic Concepts

- Characteristic Function (Membership Function)
- Notation
- Semantics and Interpretations
- Related crisp sets
- Support, Bandwidth, Core, α-level cut
- Features, Properties, and More Definitions
- Convexity, Normality
- Cardinality, Measure of Fuzziness
- MF parametric formulation
- Fuzzy Set-theoretic Operations
- Intersection, Union, Complementation
- T-norms and T-conorms
- Numerical Examples
- Fuzzy Rules and Fuzzy Reasoning
- Extension Principle and Fuzzy Relations
- Fuzzy If-Then Rules
- Fuzzy Reasoning
- Fuzzy Inference Systems
- Mamdani Fuzzy Models
- Sugeno Fuzzy Models
- Tsukamoto Fuzzy Models
- Input Space Partitioning
- Fuzzy Modeling.

The father of fuzzy logic is Lotfi Zadeh who is still there, proposed in 1965. Fuzzy logic can manipulate those kinds of data which are imprecise.

Basic definitions & terminology:

Fuzzy Number:

A fuzzy number is fuzzy subset of the universe of a numerical number that satisfies condition of normality & convexity.It is the basic type of fuzzy set.

why fuzzy is used? Why we will be learning about fuzzy? The word fuzzy means that, in general sense when we talk about the real world, our expression of the real world, the way we quantify the real world, the way we describe the real world, are not very precise.

When I ask what your height is, nobody would say or nobody would expect you to know a precise answer. If I ask a precise question, probably, you will give me your height as 5 feet 8 inches. But normally, when I see people, I would say this person is tall according to my own estimate, my own belief and my own experience; or if I ask, what the temperature is today,the normal answer people would give is, today it is very hot or hot or cool. Our expression about the world around us is always not precise. Not to be precise is exactly what is fuzzy.

Fuzzy logic is logic which is <u>not very precise</u>. Since we deal with our world with this imprecise way, naturally, the computation that involves the logic of impreciseness is much more powerful than the computation that is being carried through a precise manner, or rather precision logic based computation is inferior; not always, but in many applications, they are very inferior in terms of technological application in our day to day benefits, the normal way.

Fuzzy logic has become very popular; in particular, the Japanese sold the fuzzy logic controller, fuzzy logic chips in all kinds of house hold appliances in early 90‟s. Whether it is washing machine or the automated ticket machine, anything that you have, the usual house hold appliances, the Japanese actually made use of the fuzzy logic and hence its popularity grew.

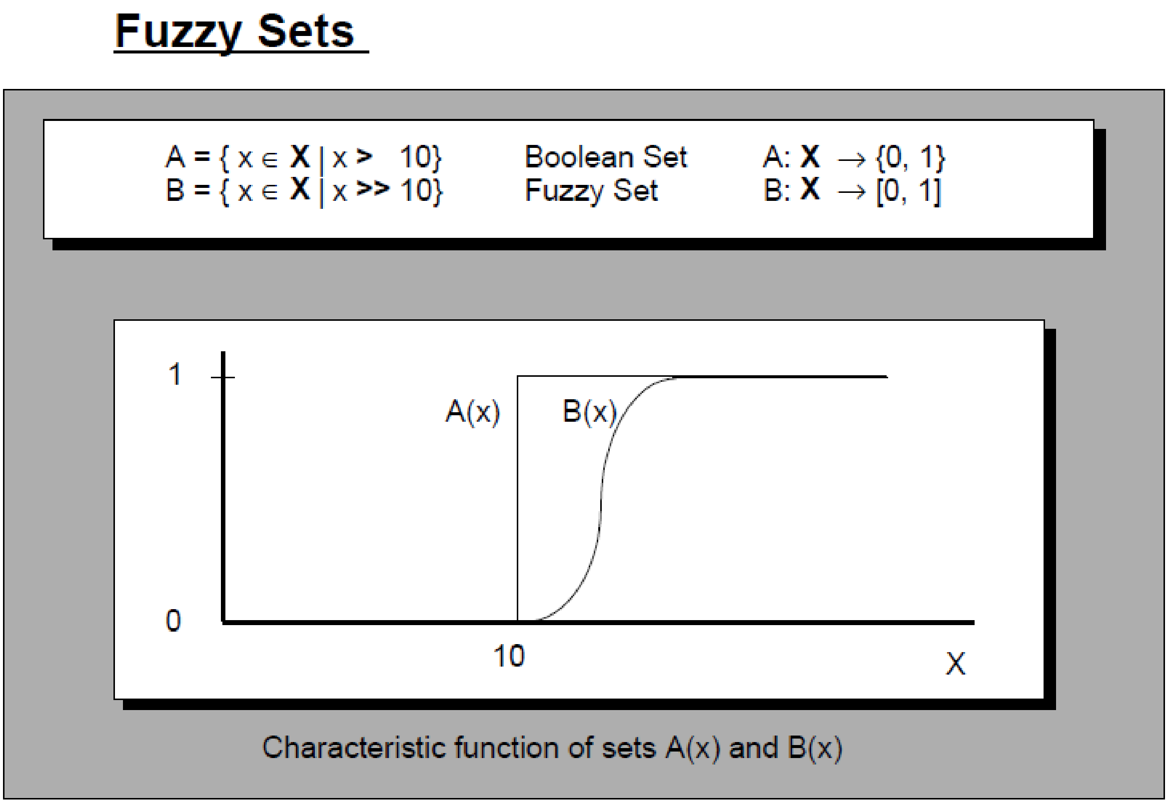


Fig. Difference in Fuzzy and crisp boundary

As fuzzy means from precision to imprecision. Here, when I say 10, I have an arrow at 10, pointing that I am exactly meaning 10 means 10.00000 very precise. When I say they are all almost 10, I do not mean only 10, rather in the peripheral 10. I can tolerate a band from minus 9 to 9, whereas if I go towards 9 or 11, I am going away from 10, the notion of 10. That is what is almost 10, that is around 10, but in a small bandwidth, I still allow certain bandwidth for 10.

This concept to be imprecise is fuzzy or to deal with the day to day data that we collect or we encounter and representing them in an imprecise manner like here almost 0, near 0, or hot, cold, or tall; if I am referring to height, tall, short medium. This kind of terminology that we normally talk or exchange among ourselves in our communication actually deals with imprecise data rather than precise data. Naturally, since our communications are imprecise, the computation resulting out of such communication language, the language which is imprecise must be associated with some logic.

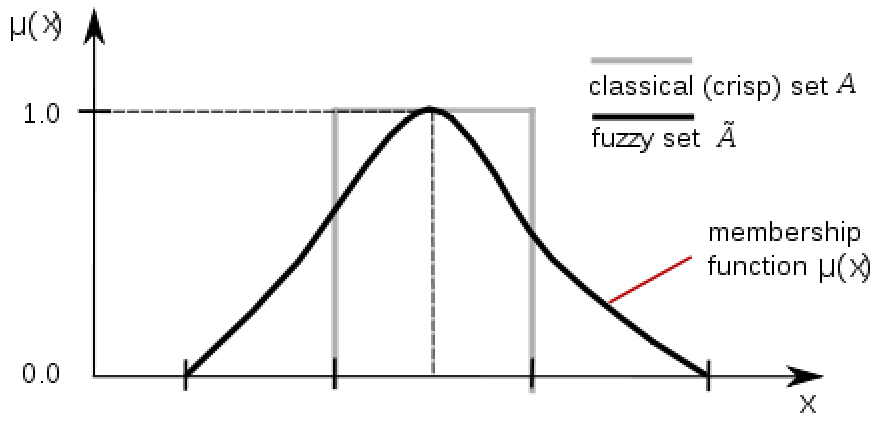


Fig. Sets: classical & fuzzy boundary

Set: A collection of objects having one or more common characteristics. For example, set of natural number, set of real numbers, members, or elements. Objects belonging to a set is represented as x belonging to A, where A is a set.

Universe of Discourse:
Defined as "a collection of objects all having the same characteristics".
Notation: U or X, and elements in the universe of discourse are: u or x

Now, we will be talking about fuzzy sets. When I talked about classical set, we had classical set of the numbers that we know, like we talked about the set of natural numbers, set of real numbers. What is the difference between a fuzzy set and a classical set or a crisp set? The difference is that the members, they belong to a set A or a specific set A or B or X or Y, whatever it is, we define them; but the degree of belonging to the set is imprecise. If I say, a universal set in natural numbers, all the natural numbers fall in this set. If I take a subset of this natural number, like in earlier case, we put 1 to 11 in one set. When I ask, whether 12 belongs to set A, the answer is no; 13 belongs to set A? The answer is no; because, in my natural number set, only 1 to 11 are placed. This is called classical set and their belongingness here is one. They all belong to this set.

But in a fuzzy set, I can have all the numbers in this set, but with a membership grade associated with it. When I say membership grade is 0 that means, they do not belong to the set, whereas a membership grade between 0 to 1, says how much this particular object may belong to the set.

The nomenclature/ Notation of a fuzzy set - how do we represent a fuzzy set there? One way is that let the elements of X be x1, x2, up to xn; then the fuzzy set A is denoted by any ofthe following nomenclature.
Mainly 2 types:

1. Numeric

2. Functional

Mostly, we
will be using either this or the first one, where you see the ordered pair x
1 μ A x1; x1 is member of A and x1 is associated with a fuzzy index and so forth, x2 and its fuzzy index, xn and its fuzzy membership. The same thing, I can also write x1 upon μ A x1.

That means x1 is the member and this is the membership. The other way is here, in the third pattern the membership is put first and in the bottom the member x1 with a membership, x2 with membership and xn with membership.
Every member x of a fuzzy set A is assigned a fuzzy index. This is the membership grade µA x in the interval of 0 to 1, which is often called as the grade of membership of x in A. In a classical set, this membership grade is either 0 or 1; it either belongs to set A or does not belong. But in a fuzzy set this answer is not precise, answer is, it is possible. It is belonging toset A with a fuzzy membership 0.9 and I say it belongs to A with a fuzzy membership 0.1; that is, when I say 0.9, more likely it belongs to set A. When I say 0.1, less likely it belongsto set A. Fuzzy sets are a set of ordered pairs given by A. The ordered pair is x, where x is a member of the set. Along with that, what is its membership grade and how likely the subject belongs to set A? That is the level we put, where x is a universal set and µx is the grade of membership of the object x in A. As we said, this membership µ.
A x lies between 0 to 1; so, more towards 1, we say more likely it belongs to A. Like if I say membership grade is 1, certainly it belongs to A.

For an example: a set of all tall people. Tall if I define, classically I would say above 6 is tall and below 6 is not tall; that is, 5.9, 5 feet 9 inches is not tall and 6.1, 6 feet 1 inch is tall. That looks very weird; it does not look nice to say that a person who is 6 feet 1 inch is tall and 5 feet 9 inches is not tall. This ambiguity that we have in terms of defining such a thing in classical set, the difficulty that we face can be easily resolved in fuzzy set. In fuzzy set, we can easily say both 6.1, 6 feet 1 inch as well as 5.9 inches as tall, but level this difference; they are tall, but with a membership grade associated with this. This is what fuzzy set is.

Membership function - a membership function µ A x is characterized by µ A that maps all the members in set x to a number between 0 to 1, where x is a real number describing an object or its attribute, X is the universe of discourse and A is a subset of X.

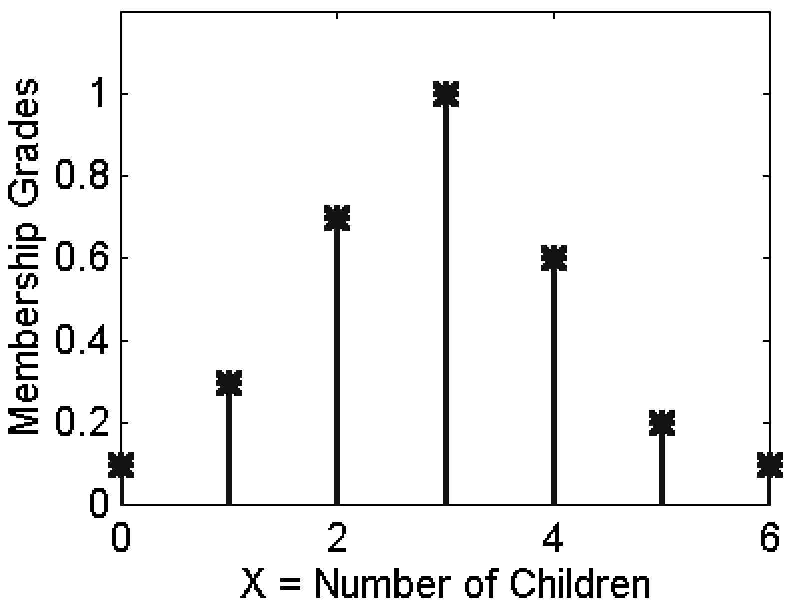


Fig. Fuzzy Sets with Discrete Universes
Fuzzy set A = “sensible number of children”
X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}--(See discrete ordered pairs)(1^{st} expression)

or

$$A = \{(x, \mu_A(x)) \mid x \in X\},$$

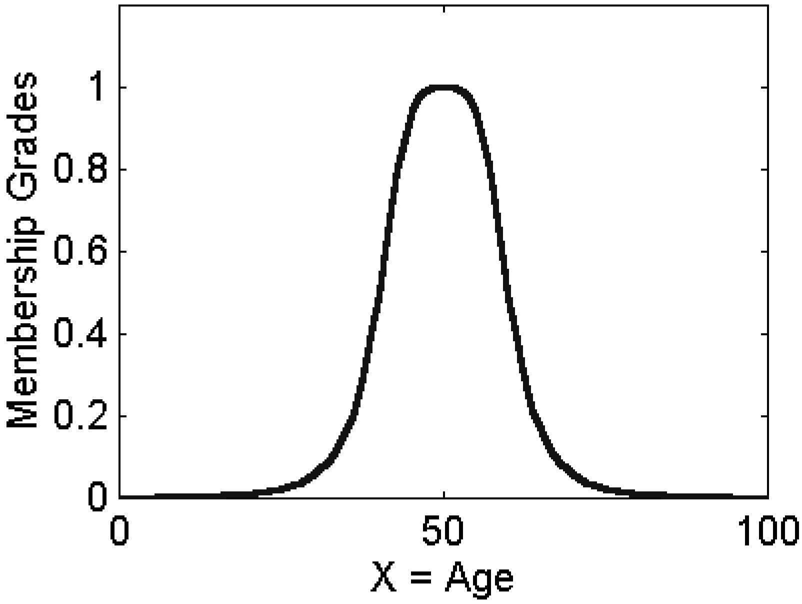


Fig. Fuzzy Set with Cont. Universe

Fuzzy set B = “about 50 years old”
X = Set of positive real numbers (continuous)
B = {(x, μB(x)) | x in X}
μB(x)=f(x)

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x-50}{10}\right)^4}.$$

(2nd expression –with function that is subjective)
3rd expression of fuzzy set:

$$A = \begin{cases} \sum_{x_i \in X} \mu_A(x_i)/x_i, & \text{if } X \text{ is a collection of discrete objects.} \\ \int_X \mu_A(x)/x, & \text{if } X \text{ is a continuous space (usually the real line } R). \end{cases}$$

$$A = 0.1/0 + 0.3/1 + 0.7/2 + 1.0/3 + 0.7/4 + 0.3/5 + 0.1/6,$$

$$B = \int_{R+} \frac{1}{1 + (\frac{x-50}{10})^4} \Big/ x,$$

Linguistic variable and linguistic values:
Linguistic variable is a variable expressed in linguistic terms e.g. “Age” that assumes various linguistic values like :middleaged, young, old. The linguistic variables are characterized by membership functions.

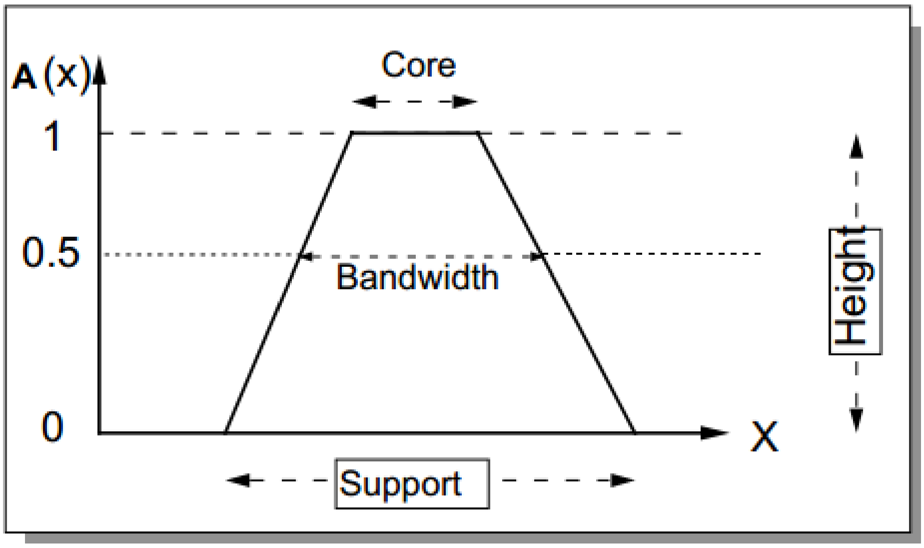


Fig. A membership function showing support, bandwidth, core, crossover points

Support:

Support of a fuzzy set A is the set of all points x in X such that μA(x)>0.

Support (A)= {x| μA(x)>0}

Core:

The core of a fuzzy set A is the set of all poits x in X such that μA(x)=1

core (A)= {x| μA(x)=1}

Normality:

A fuzzy set A is normal if its core is nonempty. Always there is at least one x with μA(x)=1 then it is normal.

Crossover point:

A cross over point in fuzzy set A is the x with μA(x)=0.5

crossover (A)= {x| μA(x)=0.5}

Bandwidth:

For a normal & convex fuzzy set

Width(A)=|x2-x1|, where x2 & x1 are crossover points.

fuzzy singleton:

A fuzzy set whose support is a single point in X with μA(x)=1 is called a fuzzy singleton.

The **α-cut** or **α-level set** of a fuzzy set A is a crisp set defined by

$$A_\alpha = \{x | \mu_A(x) \geq \alpha\}.$$

Strong α-cut or **strong α-level set** are defined similarly:

$$A'_\alpha = \{x | \mu_A(x) > \alpha\}.$$

For the set given in figure we can find equivalence & write

$$\text{support}(A) = A'_0,$$

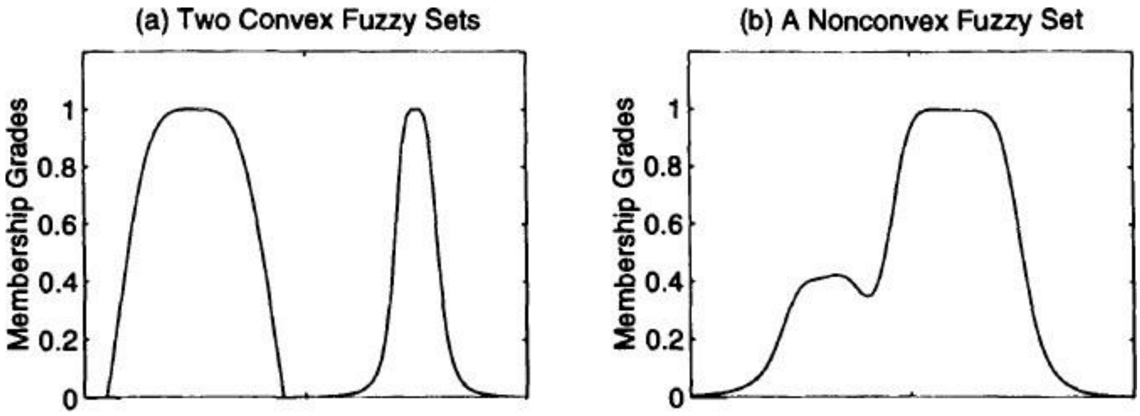
$$\text{core}(A) = A_1,$$

Convexity:

A fuzzy set A is **convex** if and only if for any $x_1,\ x_2 \in X$ and any $\lambda \in [0,1]$,

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}.$$

Alternatively, A is convex if all its α-level sets are convex.



Symmetry:

A fuzzy set is symmetric if its MF is symmetric about a certain point x=c such that,

μA(c+x)= μA(c-x) for all x in X

Comparison of the classical approach and fuzzy approach:
Let us say, consider a universal set T which stands for temperature. Temperature I can say cold, normal and hot. Naturally, these are subsets of the universal set T; the cold temperature,normal temperature and hot temperature they are all subsets of T.
The classical approach, probably, one way to define the classical set is cold. I define cold: temperature T; temperature is a member of cold set which belongs to the universal set T such that this temperature, the member temperature is between 5 degree and 15 degree centigrade.
Similarly, the member temperature belongs to normal, if it is between 15 degree centigrade and 25 degree centigrade. Similarly, the member temperature belongs to hot set when the

temperature is between 25 degree centigrade and 35 degree centigrade. As I said earlier, one should notice that 14.9 degree centigrade is cold according to this definition while 15.1 degree centigrade is normal implying the classical sets have rigid boundaries and because of this rigidity, the expression of the world or the expression of data becomes very difficult. For me, I feel or any one of us will feel very uneasy to say that 14.9 degrees centigrade is coldand 15.1 degree centigrade is normal or for that matter, 24.9 degrees centigrade is normal and25 degree or 25.1 degree centigrade is hot. That is a little weird or that is bizarre to have such an approach to categorize things into various sets.

In a fuzzy set, it is very easy to represent them here. If the temperature is around 10 degree centigrade, it is cold; temperature is around 20 degrees centigrade, it is normal and when temperature is around 30 degree centigrade it is hot. In that sense, they do not have a rigid boundary. If you say here, 25 degree centigrade, the 25 degree centigrade can be called simultaneously hot as well as normal, with a fuzzy membership grade 0.5. 25 degrees centigrade belongs to both normal as well as hot, but when I say 28 degree centigrade, this is more likely a temperature in the category of hot, whereas the 22 degree centigrade is a temperature that is more likely belonging to the set normal. This is a much nicer way to represent a set. This is how the imprecise data can be categorized in a much nicer way using fuzzy logic. This is the contrasting feature, why the fuzzy logic was introduced in the first place.

Fuzzy sets have soft boundaries. I can say cold from almost 0 degree centigrade to 20 degree centigrade. If 10 degree has a membership grade 1 and as I move away from 10 degree in both directions, I lose the membership grade. The membership grade reduces from 1 to 0 here, and in this direction also from 1 to 0. The temperature, As I go, my membership grade reduces; I enter into a different set simultaneously and that is normal. You can easily see, liketemperature 12, 13, 14, 15 all belong to both categories cold as well as normal, but each member is associated with a membership grade; this is very important.

In a classical set, there are members in a set. Here, there are members in a set associated with a fuzzy index or membership function.

LECTURE-3

Parameterization of Membership Function:

Once we talk about each member in a fuzzy set associated with membership function, you must know how to characterize this membership function. The parameters are adjusted to finetune a fuzzy inference system to achieve desired I/O mapping. The membership functions given here are one- dimensional. 2 dimensional MFs can be formed by cylindrical extension from these basic MFs.

$$\text{triangle}(x; a, b, c) = \begin{cases} 0, & x \le a. \\ \dfrac{x-a}{b-a}, & a \le x \le b. \\ \dfrac{c-x}{c-b}, & b \le x \le c. \\ 0, & c \le x. \end{cases}$$

Where a<b<c & that are x coordinates of the corners of triangular MF

$$
\text{trapezoid}(x;a,b,c,d) = \begin{cases} 0, & x \le a. \\ \dfrac{x-a}{b-a}, & a \le x \le b. \\ 1, & b \le x \le c. \\ \dfrac{d-x}{d-c}, & c \le x \le d. \\ 0, & d \le x. \end{cases}
$$

Where $a<b<c<d$ & that are x coordinates of the corners of trapezoidal MF

$$
\text{bell}(x;a,b,c) = \frac{1}{1+\left|\dfrac{x-c}{a}\right|^{2b}},
$$

Where c is the centre & a is adjusted to vary the width of MF, b controls slope at crossover points.
Bell membership function is also termed as Cauchy MF.

$$
\text{gaussian}(x;c,\sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.
$$

Where c is the centre & σ is the width of MF.

Left-Right MF:

$$
\text{LR}(x;c,\alpha,\beta) = \begin{cases} F_L\left(\dfrac{c-x}{\alpha}\right), & x \le c. \\ F_R\left(\dfrac{x-c}{\beta}\right), & x \ge c, \end{cases}
$$

Sigmoidal MF:

A **sigmoidal MF** is defined by

$$
\text{sig}(x;a,c) = \frac{1}{1+\exp[-a(x-c)]},
$$

where a controls the slope at the crossover point $x = c$.

It can be open left or open right depending on sign of a.

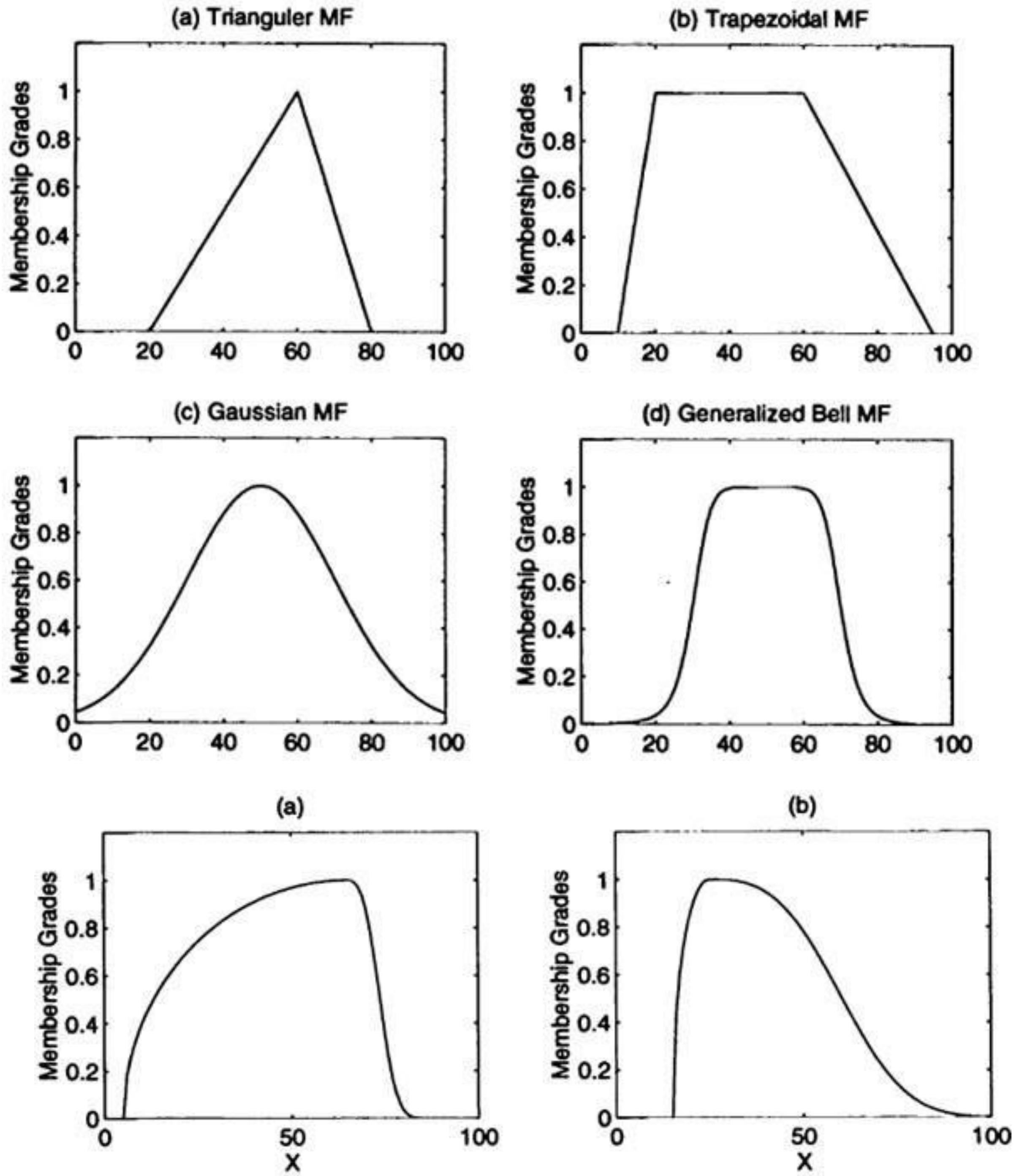


Fig. Membership functions a. Triangle b. Trapezoidal c. Gaussian d. Bell, e. Left f. Right

LECTURE-4

Fuzzy set operations:

The main features of operation on fuzzy set are that unlike conventional sets, operations on fuzzy sets are usually described with reference to membership function. When I say operation, I do not do with the member itself, but I manipulate. When I say operation, I manipulate the membership of the members in a set; members are not manipulated, rather the membership function of the member is manipulated. This is very important; that is, x and μ(x). In classical set what is manipulated is x.

If I say, x is 1 In classical set when I say x is 1 then, I would say 1 minus x is 0. In this, the manipulation concerns with the member; whereas any kind of manipulation in fuzzy set does not involve with x; rather it involves μx.

Containment or subset:

Fuzzy set A is **contained** in fuzzy set B (or, equivalently, A is a **subset** of B, or A is smaller than or equal to B) if and only if $\mu_A(x) \le \mu_B(x)$ for all x. In symbols,

$$A \subseteq B \Longleftrightarrow \mu_A(x) \le \mu_B(x)$$

Three common operations: intersection which we say is the minimum function, union, which we say is the maximum function and then fuzzy complementation

Standard fuzzy operations:

Intersection(Conjunction)or T-norm:

We can easily see that, the membership of A (green) intersection B(red) in fig. is all the members that belongs to, that is common between A and B. Their membership will follow these (blue) curves. There are two things we are doing. We have 2 sets. One is set A and the other is set B. Classically, what we see is the common members between A and B. We are not only seeing the common members, here we are also seeing, what is their membership function.

$$\mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x).$$

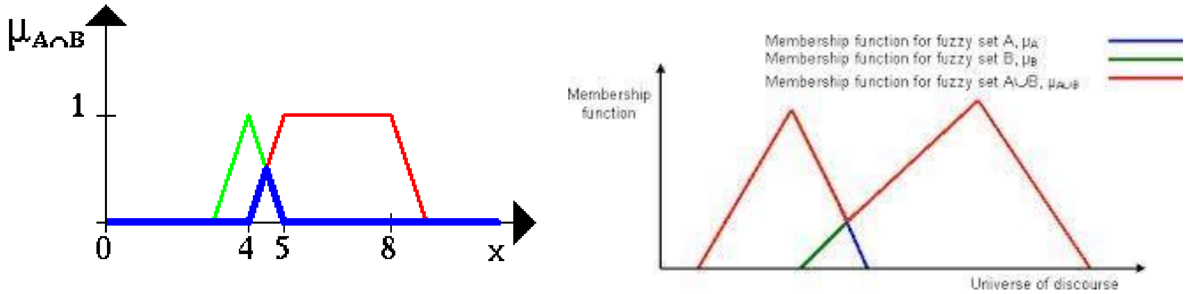


Fig. Fuzzy set operations intersection & union

The membership function is computed minimum; that is, μA intersection B is minimum of μA x and μ B x. That is the membership function. When there is a common member between A and B, the membership function wherever is minimum that is retained and the other one is thrown away. The member is retained; what is changing is the membership function.

Union(Disjunction) or T-co-norm or S-norm:

That is the meaning of these two curves that we have and then we are trying to find out what the fuzzy union is. I have to find out In this the members are both belonging to A and B. But their membership is maximum of both. if I have common members. I have set A and I have set B; A union B is my union set. If x belongs to A and x belongs to B, then x also belongs to A union B. But in fuzzy set, here this is μ_A x and here it is μ_B x and in this case, this is maximum of μ_A x and μ_B x; the membership function. That is the way it is assigned.

$$\mu_C(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x).$$

This candidate, when it comes to A union B take these two values of membership, find the maximum which is 0.1 and assign here, which is 0.1. This is, μ union $_B$ is 0.1. This is the meaning. This is a very important operation that we do. When we have two different fuzzy sets, the operations are classical. The manipulation is among the membership functions; otherwise, the notion of the classical fuzzy operation also remains intact, except that the associated fuzzy membership gets changed.

Complement(Negation):

now it is fuzzy complementation. What is complement? This one, this particular triangular function is my set R(red); fuzzy set R. The complement is like this; just inverse (blue). What is 1 minus μ_A x; meaning 1 minus μ_A x.

$$\mu_{\overline{A}}(x) = 1 - \mu_A(x).$$

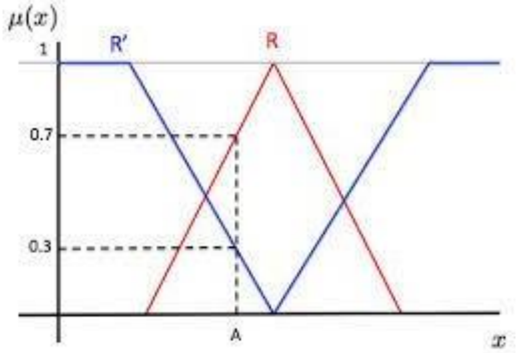


Fig. Complement of fuzzy set

What is seen that the members remain intact in the set A, whereas the associated membership functions got changed.

The other operations that we know for classical sets like De Morgan‟s law, the difference also can be used for the sets like De Morgan‟s law.

Properties/ identities of fuzzy sets:

They are <u>commutative</u>. A union B is B union A; A intersection B is B intersection A. It is like classical sets; fuzzy sets equally hold.

<u>Associativity</u>; A union B union C is A union B union C. Similarly, A union bracket B union C is A intersection B intersection C is A intersection B combined with intersection C.

Distributivity: you can easily see that A union B intersection C is A union B intersection A union C which is here. Similarly, here A intersection B union A intersection C. So, this is distributivity.

Idempotency which is A union A is A and A intersection A is A.

Identity: A union null set is A, A intersection universal set is A, A intersection null set is null and A union universal set is universal set X; here, X represents universal set.

The next step in establishing a complete system of fuzzy logic is to define the operations of EMPTY, EQUAL, COMPLEMENT (NOT), CONTAINMENT, UNION (OR), and INTERSECTION (AND). Before we can do this rigorously, we must state some formal definitions:

Definition 1: Let X be some set of objects, with elements noted as x. Thus, X = {x}.

Definition 2: A fuzzy set A in X is characterized by a membership function mA(x) which maps each point in X onto the real interval [0.0, 1.0]. As mA(x) approaches 1.0, the "grade of membership" of x in A increases.

Definition 3: A is EMPTY iff for all x, μA(x) = 0.0.

Definition 4: A = B iff for all x: μA(x) = μB(x) [or, μA = μB].

Definition 5: μA' = 1 - μA.

Definition 6: A is CONTAINED in B iff μA <= μB.

Definition 7: C = A UNION B, where: μC(x) = MAX(μA(x), μB(x)).

Definition 8: C = A INTERSECTION B where: μC(x) = MIN(μA(x), μB(x)).

Difference probability & fuzzy operations:

It is important to note the last two operations, UNION (OR) and INTERSECTION (AND), which represent the clearest point of departure from a probabilistic theory for sets to fuzzy sets. Operationally, the differences are as follows:

For independent events, the probabilistic operation for AND is multiplication, which (it can be argued) is counterintuitive for fuzzy systems. For example, let us presume that x = Bob, S is the fuzzy set of smart people, and T is the fuzzy set of tall people. Then, if μS(x) = 0.90 and μT(x) = 0.90, the probabilistic result would be:

μS(x) * μT(x) = 0.81

whereas the fuzzy result would be:

MIN(μS(x), μT(x)) = 0.90

The probabilistic calculation yields a result that is lower than either of the two initial values, which when viewed as "the chance of knowing" makes good sense. However, in fuzzy terms the two membership functions would read something like "Bob is very smart" and "Bob is very tall." If we presume for the sake of argument that "very" is a stronger term than "quite," and that we would correlate "quite" with the value 0.81, then the semantic difference becomes obvious. The probabilistic calculation would yield the statement If Bob is very smart, and Bob is very tall, then Bob is a quite tall, smart person. The fuzzy calculation, however, would yield If Bob is very smart, and Bob is very tall, then Bob is a very tall, smart person.

Another problem arises as we incorporate more factors into our equations (such as the fuzzy set of heavy people, etc.). We find that the ultimate result of a series of AND's approaches 0.0, even if all factors are initially high. Fuzzy theorists argue that this is wrong: that five factors of the value 0.90 (let us say, "very") AND'ed together, should yield a value of 0.90 (again, "very"), not 0.59 (perhaps equivalent to "somewhat").

Similarly, the probabilistic version of A OR B is (A+B - A*B), which approaches 1.0 as additional factors are considered. Fuzzy theorists argue that a sting of low membership grades should not produce a high membership grade instead, the limit of the resulting membership grade should be the strongest membership value in the collection.
The skeptical observer will note that the assignment of values to linguistic meanings (such as 0.90 to "very") and vice versa, is a most imprecise operation. Fuzzy systems, it should be noted, lay no claim to establishing a formal procedure for assignments at this level; in fact, the only argument for a particular assignment is its intuitive strength. What fuzzy logic does propose is to establish a formal method of operating on these values, once the primitives have been established.

Hedges :

Another important feature of fuzzy systems is the ability to define "hedges," or modifier of fuzzy values. These operations are provided in an effort to maintain close ties to natural language, and to allow for the generation of fuzzy statements through mathematical calculations. As such, the initial definition of hedges and operations upon them will be quite a subjective process and may vary from one project to another. Nonetheless, the system ultimately derived operates with the same formality as classic logic. The simplest example is in which one transforms the statement "Jane is old" to "Jane is very old." The hedge "very" is usually defined as follows:

µ"very"A(x) = µA(x)^2

Thus, if mOLD(Jane) = 0.8, then mVERYOLD(Jane) = 0.64.

Other common hedges are "more or less" [typically SQRT(µA(x))], "somewhat," "rather," "sort of," and so on. Again, their definition is entirely subjective, but their operation is consistent: they serve to transform membership/truth values in a systematic manner accordingto standard mathematical functions.

$$\mathrm{CON}(A) = A^2,$$

$$\mathrm{DIL}(A) = A^{0.5}.$$

Cartesian Product & Co-product:

Let A & B be fuzzy sets in X & Y respectively, then Cartesian product of A & B is a fuzzy set in the product space XxY with the membership function

$$\mu_{A\times B}(x,y) = \min(\mu_A(x), \mu_B(y)).$$

Similarly, Cartesian co-product A+B is a fuzzy set

$$\mu_{A+B}(x,y) = \max(\mu_A(x), \mu_B(y)).$$

Both Product & Co-product are characterized by 2- dimensional MFs.

LECTURE-5

Fuzzy Extension Principle:

Consider a function *y* = *f* (*x*).
If we known *x* it is possible to determine *y*.
Is it possible to extend this mapping when the input, *x*, is a fuzzy value.

The extension principle developed by Zadeh (1975) and later by Yager (1986) establishes how to extend the domain of a function on a fuzzy sets.

Suppose that *f* is a function from *X* to *Y* and *A* is a fuzzy set on *X* defined as
A = µA(*x*1)/*x*1 + µA(*x*2)/*x*2 + . . . + µA(*x*n)/*x*n.
The extension principle states that the image of fuzzy set *A* under the mapping *f* (.) can be expressed as a fuzzy set *B* defined as
B = *f* (*A*) = µA(*x*1)/*y*1 + µA(*x*2)/*y*2 + . . . + µA(*x*n)/*y*n
where *yi* = *f* (*x*i)
If *f* (.) is a many-to-one mapping, then, for instance, there may exist *x*1, *x*2 ∈ *X*, *x*1 6= *x*2, such that *f* (*x*1) = *f* (*x*2) = *y*_, *y*_ ∈ *Y* . The membership degree at *y* = *y*∗ is the maximum of the membership degrees at *x*1 and *x*2 more generally, we have µB(*y*_) = maxy=f (xi) µA(*x*)

A point to point mapping from a set A to B through a function is possible. If it is many to one for two x in A then the membership function value in set B is calculated for f(x) as max value of MF.

Let

$$A = 0.1/-2 + 0.4/-1 + 0.8/0 + 0.9/1 + 0.3/2$$

and

$$f(x) = x^2 - 3.$$

Upon applying the extension principle, we have

$$\begin{aligned} B &= 0.1/1 + 0.4/-2 + 0.8/-3 + 0.9/-2 + 0.3/1 \\ &= 0.8/-3 + (0.4 \vee 0.9)/-2 + (0.1 \vee 0.3)/1 \\ &= 0.8/-3 + 0.9/-2 + 0.3/1, \end{aligned}$$

Fuzzy Relation:

CRISP MAPPINGS:

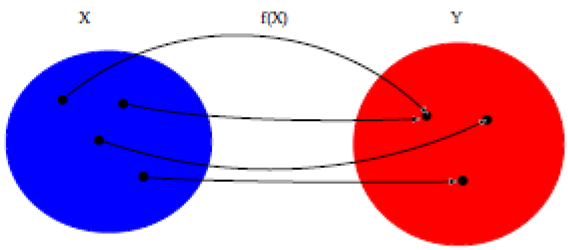


Fig. Mapping a relation

Consider the Universe *X* = {−2,−1, 0, 1, 2}
Consider the set *A* = {0, 1}
Using the Zadeh notation *A* = { 0/ −2 + 0/−1 + 1/ 0 + 1/ 1 + 0/ 2}
Consider the mapping *y* = |4*x*| + 2
What is the resulting set *B* on the Universe *Y* = {2, 6, 10}

It is possible to achieve the results using a relation that express the mapping $y = |4x| + 2$.
Lets $X = \{-2, -1, 0, 1, 2\}$.
Lets $Y = \{0, 1, 2, \ldots, 9, 10\}$
The relation

$$R = \begin{array}{c} \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \left[\begin{array}{ccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$B = A \circ R$

$A = \{\frac{0}{-2} + \frac{0}{-1} + \frac{1}{0} + \frac{1}{1} + \frac{0}{2}\}$ or more conveniently $A = \{0, 0, 1, 1, 0\}$

Using $\chi_B(y) = \bigvee_{x \in X}(\chi_A(x) \wedge \chi_R(x, y))$
we find

$$\chi_B(y) = \begin{cases} 1, & \text{for } y = 2, 6 \\ 0, & \text{otherwise} \end{cases}$$

.

Or

$$B = \left\{\frac{0}{0} + \frac{0}{1} + \frac{1}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5} + \frac{1}{6} + \frac{0}{7} + \frac{0}{8} + \frac{0}{9} + \frac{0}{10}\right\}$$

Fuzzy Mappings:

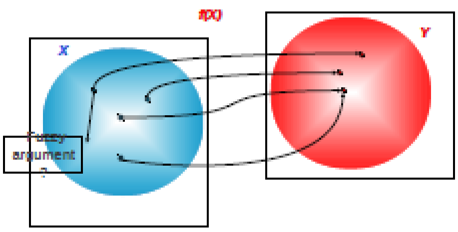


Fig. Fuzzy arguments mapping

Consider two universes of discourse X and Y and a function $y = f(x)$.
Suppose that elements in universe X form a fuzzy set A.
What is the image (defined as B) of A on Y under the mapping f?
Similarly to the crisp definition, B is obtained as

$$\mu_B(y) = \mu_{f(A)}(y) = \bigvee_{y=f(x)} \mu_A(x)$$

Fuzzy vector is a convenient shorthand for calculations that use matrix relations.
Fuzzy vector is a vector containing only the fuzzy membership values.
Consider the fuzzy set:

$$B = \left\{\frac{0}{0} + \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.9}{5} + \frac{1}{6} + \frac{0}{7} + \frac{0}{8} + \frac{0}{9} + \frac{0}{10}\right\}$$

The fuzzy set B may be represented by the fuzzy vector b:

$$b = \left\{0, 0.2, 0.3, 0.5, 0.7, 0.9, 1, 0, 0, 0, 0\right\}$$

Now, we will be talking about fuzzy relation. If x and y are two universal sets, the fuzzy sets, the fuzzy relation R x y is given. As this is all ordered pair, μR x y up on x y for all x y, belonging to the Cartesian space x, you associate μ R x y with each ordered pair.
What is the difference between fuzzy and crisp relation? In fuzzy this is missing, where μ R x y is a number in 0 and 1. μR x y is a number between 0 and 1. This is the difference between crisp relation and fuzzy relation. In crisp relation, it was either 0 or 1. It is either completely connected or not connected, but in case of fuzzy, connection is a degree; that is, it is from 0 to 1.

Let X and Y be two universes of discourse. Then

$$\mathcal{R} = \{(\ (x,y),\ \mu_{\mathcal{R}}(x,y)\ ) \mid (x,y) \in X \times Y\}$$

The example is, let x equal to 1 2 3. Then x has three members, y has two members 1 and 2. If the membership function associated with each ordered pair is given by this e to the power minus x minus y whole squared. I is seen that this is the kind of membership function that is used to know, how close is the members of y are from members of x. Because, if I relate from 1 to 1 using this, then you can see 1 minus 1 is 0 that is 1 and 1 very close to each other; whereas, 2 and 1 is little far and 3 1 one is further far. This is a kind of relationship we are looking between these two sets.
Let us derive fuzzy relation. If this is the membership function, fuzzy relation is of course all the ordered pairs. We have to find out 111 2 2 1 2 2 3 1 and 3 2. These are all the sets of ordered pairs and associated membership functions. You just compute e to the power minus xminus y whole square. Here, 1 1 1 minus 1 whole square, 1 2 1 minus 2 whole square, 2 1 2 minus 1 whole square, 2 two 2 minus 2 whole square, 3 1 3 minus 1 whole square, 3 2 3 minus 2 whole square and if you compute them, you find 1 0.4 3 0.4 3 1 0.1 6 0.4 3. This is your membership function. This is one way to find relation.
Normally, I know, it is easier to express the relation in terms of a matrix instead of this continuum fashion, where each ordered pair is associated with membership function. It is easier to appreciate the relation by simply representing them in terms of matrix. How do we do that? This is my x 1 2 3 y is 1 21 the membership function associated was 1 1 2 membership is 0.4 3 2 1 0.4 3 2 2 1 3 1 0.1 6 and 3 2 is 0.4 3 that you can easily verify here 1 3 0.4 3 0.1 6 and 1.
The membership function describes the closeness between set x and y. It is obvious that higher value implies stronger relations. What is the stronger relation? It is between 1 and 1, and they are very close to each other, and 2 and 2; they are very close to each other. Closeness between 2 and
2, between 1 and 1 is actually 1 and 1. They are very close to each other; similarly, 2 and 2. If I simply say numerical closeness, then 2 and 2 are the closest, and 1 and 1 are the closest. That is how these are the closest. Higher value implies stronger relations.

This is a formal definition of fuzzy relation; it is a fuzzy set defined in the Cartesian product of crisp sets; crisp sets x1 x2 until xn. A fuzzy relation R is defined as μR upon x1 to xn, where x1 to xn belongs to the Cartesian product space of x1 until xn; whereas, this μR the fuzzy membership associated is a number between 0 and 1.

LECTURE-6

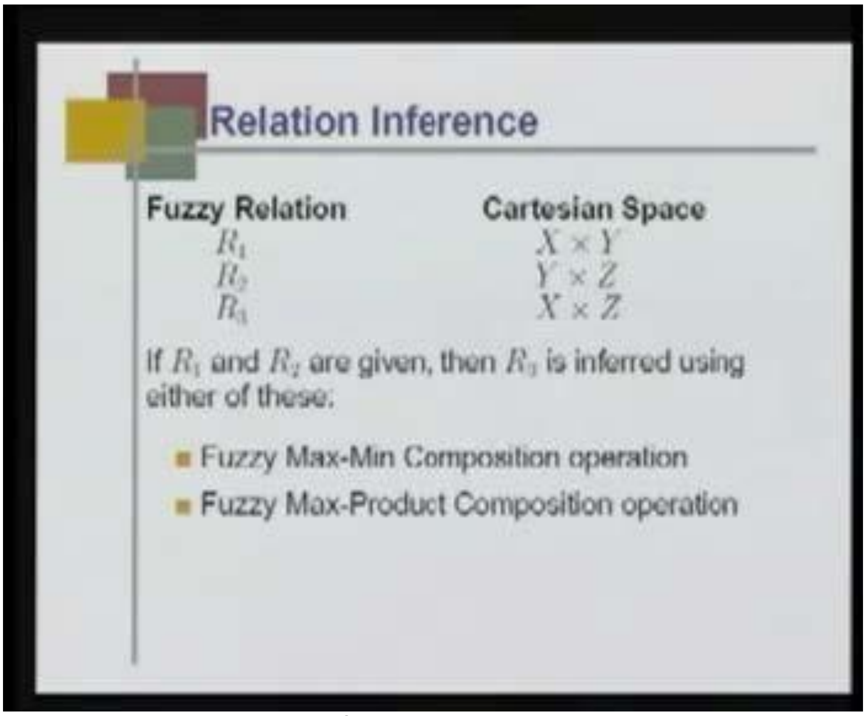


Fig. Inferring Fuzzy relation

Max-min composition or Max-min product:

Let $\mathcal{R}_1$ and $\mathcal{R}_2$ be two fuzzy relations defined on $X \times Y$ and $Y \times Z$, respecti
The **max-min composition** of $\mathcal{R}_1$ and $\mathcal{R}_2$ is a fuzzy set defined by

$$\mathcal{R}_1 \circ \mathcal{R}_2 = \{[(x, z),\ \max_y\ \min(\mu_{\mathcal{R}_1}(x, y), \mu_{\mathcal{R}_2}(y, z))] | x \in X, y \in Y, z \in Z\},$$

or, equivalently,

$$\begin{aligned} \mu_{\mathcal{R}_1 \circ \mathcal{R}_2}(x, z) &= \max_y\ \min[\mu_{\mathcal{R}_1}(x, y), \mu_{\mathcal{R}_2}(y, z)] \\ &= \vee_y\ [\mu_{\mathcal{R}_1}(x, y) \wedge \mu_{\mathcal{R}_2}(y, z)], \end{aligned}$$

It is a sort of matrix multiplication but memberships are not multiplied.

We will now explain, max min composition operation using an example that makes things much more clear. This is my matrix, relational matrix R1 relating x and y and R2 relating y and z. I have to find out the relational matrix from x to z using fuzzy rule of composition. We normally write R3 is R1 composition R2. Using max min composition, how do we compute R3?

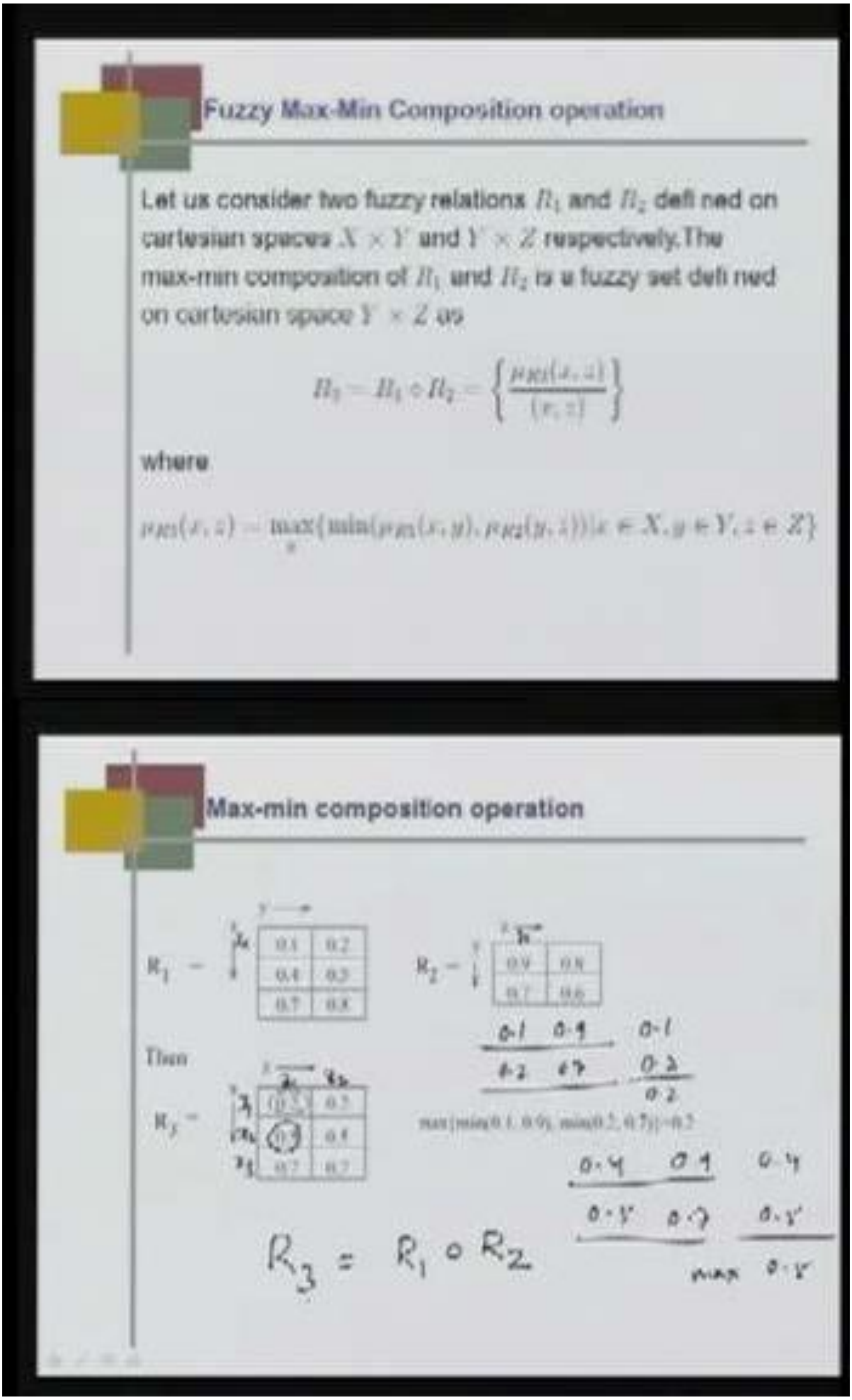


Fig. Example Max-min composition or Max-min product

I want to now build a relationship between R1 & R2. Membership associated with x1 is 0.1 and z1 is 0.9. Let me put it very precise, x1 x2 x3 z1 and z2; if you look at what we will be doing here, This is my x1 row and this is my z1 column. What I do, x1 row and z1 column; I put them parallel and find out what is minimum. Here, minimum is 0.1 and here minimum is 0.2. After that, I find out what is the maximum, which is 0.2. This is what maximum of minimum 0.1. 0.9 is minimum 0.2. 0.7 is 0.2. This is how we found out. The easiest way if I want to find out is this one; this x1 x2 and z1. x2 means this row which is 0.4 and 0.5 and x2 and z1. z1 is again 0.9 and 0.7. I will find out. Minimum here is 0.4, minimum here is 0.5 and maximum here is 0.5. You get this 0.5.Similarly, we can compute all the elements in R3 using a max min composition operation. As usual, any max min composition can follow certain properties, associative and distributive over union. That is P fuzzy composition Q union R is P composition Q union P composition R.

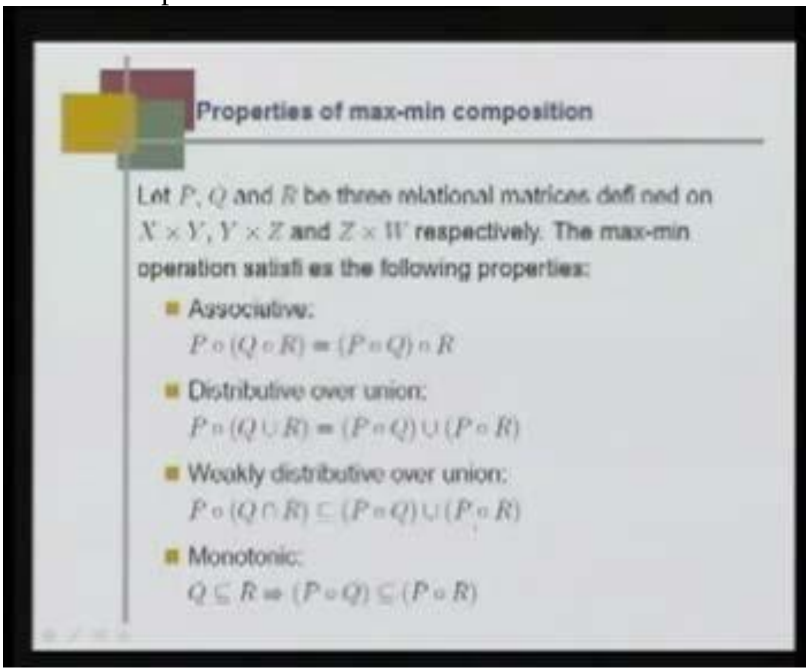


Fig. Properties of max-min composition

Similarly, weekly distributed over union is P composition, Q intersection, R is a subset of P composition. Q union P composition R monotonic Q is a subset of R implies that, P composition Q is a subset of P composition R.

Max-product composition:

$$\mu_{\mathcal{R}_1 \circ \mathcal{R}_2}(x,z) = \max_y \left[\mu_{\mathcal{R}_1}(x,y)\mu_{\mathcal{R}_2}(y,z)\right].$$

Now, again, the same example we have taken R
1, R2 and R3. Now, I want to find out from R1 and R2, what R3 using max product composition is.

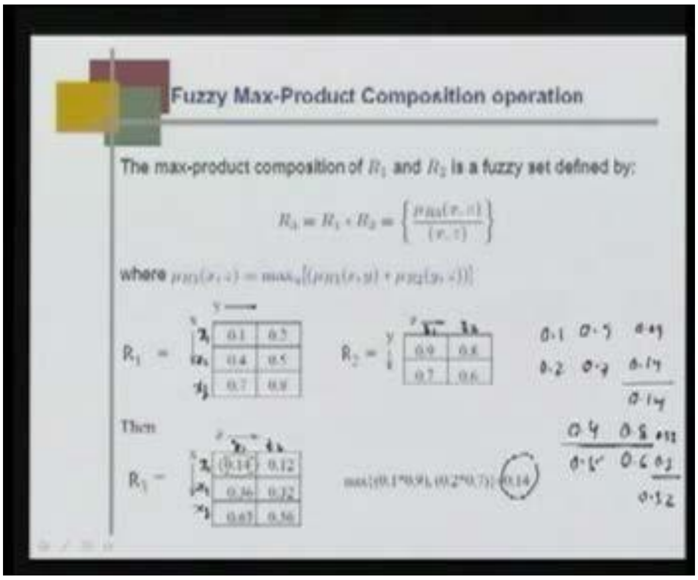


Fig. Example Max-product composition

Let us say, this is x1 x2 x3 z1 z2 z1 z2 and this is x1 x2 x3 for x1. I take this row which is 0.1 0.2 and finding the relation the fuzzy membership associate x1 and z1. I take the column from z1 which is 0.9 0.7 and I multiply them here 0.1 0.9 is point 0 9 0.2 0.7 is 0.1 4 and find out what is the maximum. This is the maximum 0.1 4.

I take another example. Let us find out the relationship between x2 and z2; for x2 the row is 0.5 and z2 the column is 0.8 0.6. Corresponding to this, if I multiply I get 0.4 0.8 is 0.3 2 0.6 is 0.3. Maximum is 0.3 2. This is 0.4 3 0.3 2. This is where it is 0.1. The answer is here, the R3 and if I go back, if I look, R3 here is different.

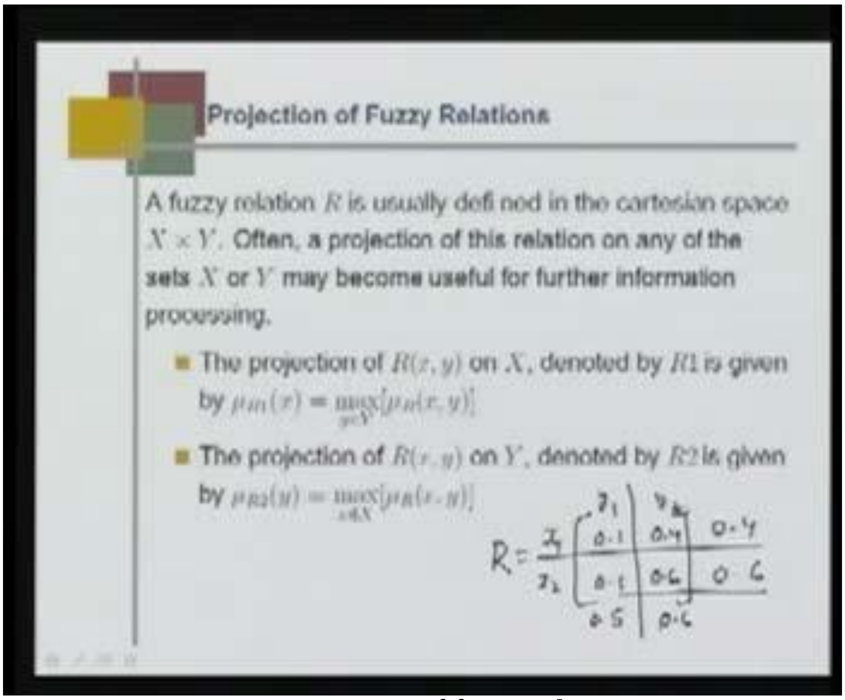


Fig. Projection of fuzzy relation

Projection of fuzzy relation:

A fuzzy relation R is usually defined in the Cartesian space x and x and y. Often a projection of this relation on any of the sets x or y, may become useful for further information processing.

The projection of R x y on x denoted by R 1 is given by µ R 1 x is maximum. So, y belongs to y µR x y. The meaning is that if I have R, this is x1 and x2 and this is y1 and y2, and this is 0.1 0.4 and this is 0.5 0.6. If these are the membership functions associated with x1 y1 x2 y2 is 0.4 x2 y1 is 0.5 x2 y2 is 0.6.projection, which means for x projection, I find out what the maximum is. Overall, y in this case maximum is 0.4 and for x2 the max maximum projection is if I took it here, 0.6. Similarly, if I make projection of R, x, y over x, what is the maximum? This is 0.5 and this is 0.6. This is called x projection and y projection of a relationmatrix R.

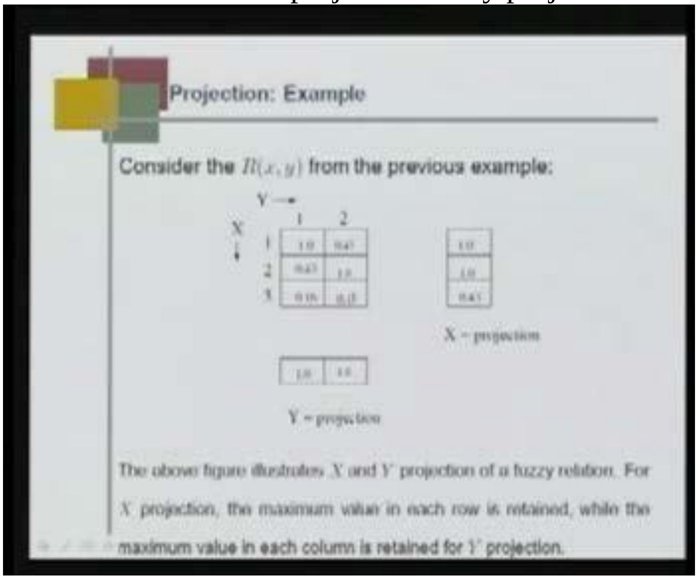


Fig. Example of projection

We repeat another example. We have x as 3 components 1 2 3, y has 2 components 1 and 2. This is the previous example that we had 1 0.4 3 0.4 3 1 0.1 6 0.4 3. x projection would be 1 3 maximum 1 0.4 3 1 maximum 1 0.1 6 0.4 3 maximum 0.4 3. Above figure illustrates x and y projection of fuzzy relation. For x projection, the maximum value in each row is retained. What is the maximum value in each row? Here, x projection maximum value in each row is retained, while the maximum value in each column is retained for y projection.

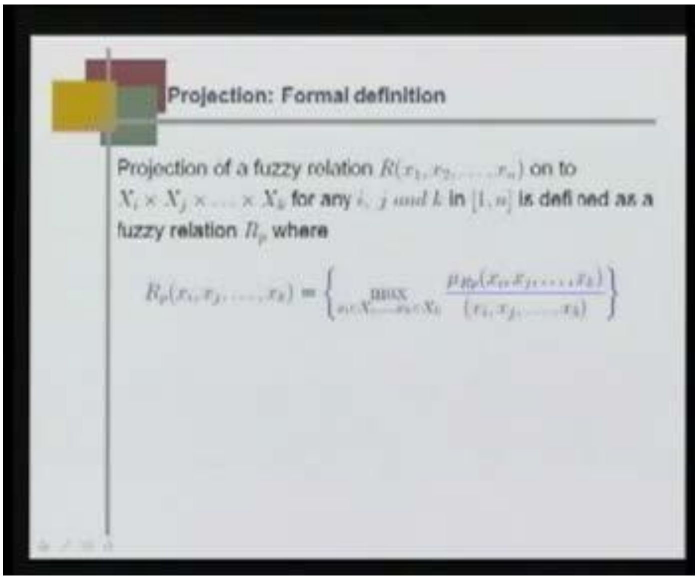


Fig. Definition of projection

This is our formal definition of a fuzzy relation, projection of a fuzzy relation R on to any of its set in the Cartesian product space; that is in the Cartesian product space. This is our Cartesian product space and for that, we can map this one to any of these i or j or k; whatever

it is, for any value, then is defined as a fuzzy relation Rp, where Rp is defined as maximum over Xi until Xk, where this is our Xi Xj Xk and this is µRp.
First, we talked about fuzzy relation projection of fuzzy relation. Once we have projection of fuzzy relation, we can extend the projection to again infer what should be the relation. This kind of technique may be useful in coding the information, where we have a huge number of information and we want to transfer such a kind of projection and from projection to extension would be beneficial for coding operation.

The crisp relation and fuzzy relation:
the difference is that in crisp relation; the index is either 0 or 1 that is, either complete relation or no relation. But in fuzzy the membership grade is either 0 or 1; Whereas, in fuzzy the relation has a grade from 0 to 1. Fuzzy composition rule; max min composition max product composition unlike in crisp relation, where both max min and max product gives you the same answer; whereas in fuzzy composition, max min and max product will give two different answers .

LECTURE-7

Fuzzy If-then rules:
A **fuzzy if-then rule** (also known as **fuzzy rule**, **fuzzy implication**, or **fuzzy conditional statement**) assumes the form

If x is A then y is B

"x is A" is antecedent or premise which tells the fact

"y is B" is consequence or conclusion
The whole statement is the rule.

Eg. If tomato is red then it is ripe.

These if then rules are the base of fuzzy reasoning.

If then rules are of different types:

1. Single rule with single antecedent
2. Single rule with multiple antecedent
3. Multiple with multiple antecedent

Steps of Fuzzy reasoning:
Shown in fig. For 2 rules what will be the consequent MF after aggregation
1. Degree of compatibility
2. Firing strength
3. Qualified consequent MF
4. Aggregate all qualified consequent MFs to obtain an overall MF

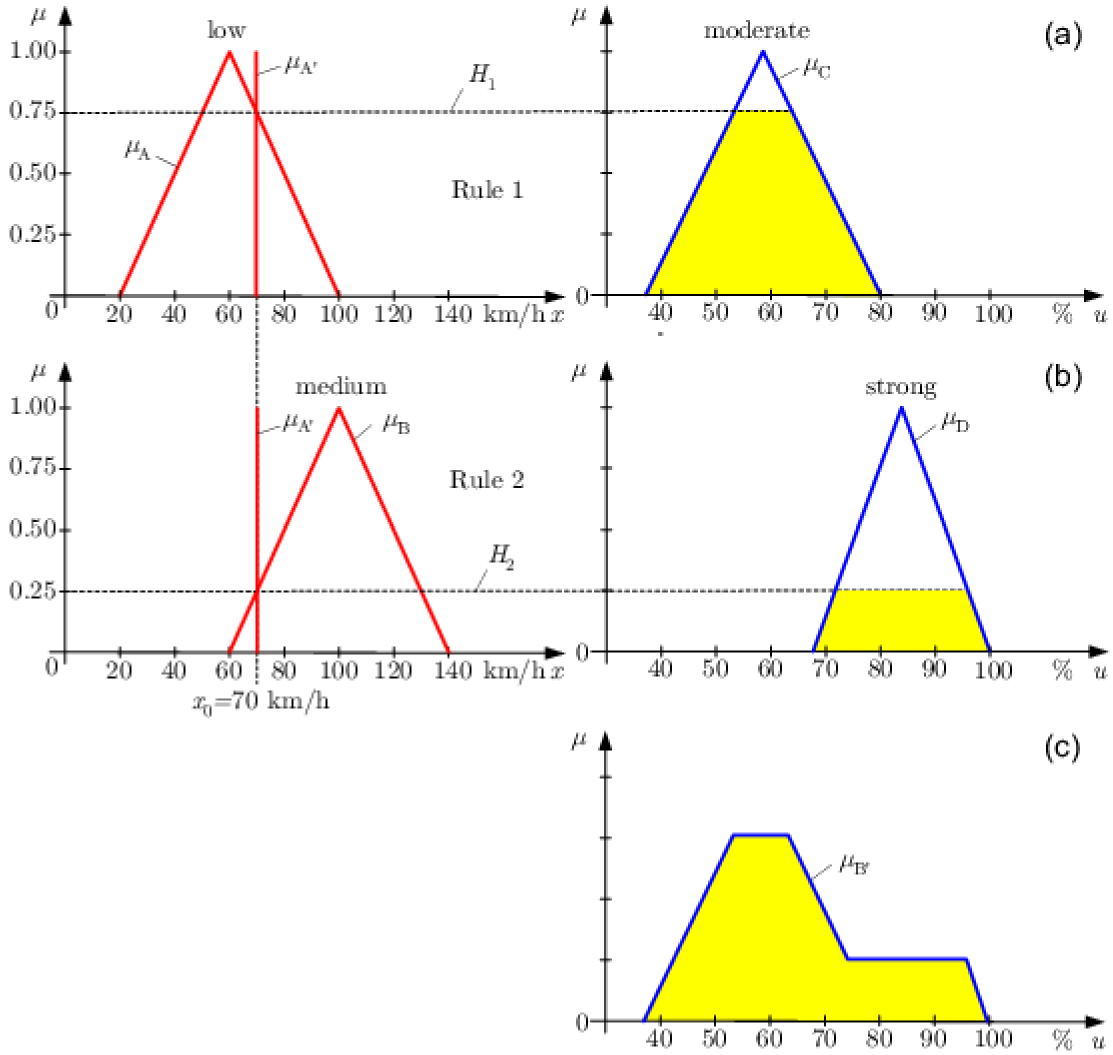


Fig. Fuzzy reasoning, deriving output

FUZZY MODELLING:

Fuzzy Inferencing

The process of fuzzy reasoning is incorporated into what is called a Fuzzy Inferencing System. It is comprised of three steps that process the system inputs to the appropriate systemoutputs. These steps are 1) Fuzzification, 2) Rule Evaluation, and 3) Defuzzification. The system is illustrated in the following figure.

Each step of fuzzy inferencing is described in the following sections.

Fuzzification

Fuzzification is the first step in the fuzzy inferencing process. This involves a domain transformation where crisp inputs are transformed into fuzzy inputs. Crisp inputs are exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.. Each crisp input that is to be processed by the FIU has its own group of membership functions or sets to which they are transformed. This group of membership functions exists within a universe of discourse that holds all relevant values that the crisp input can possess. The following shows the structure of membership functions within a universe of discourse for a crisp input.

where:
degree of membership: degree to which a crisp value is compatible to a membership function, value from 0 to 1, also known as truth value or fuzzy input.
membership function, MF: defines a fuzzy set by mapping crisp values from its domain to the sets associated degree of membership.

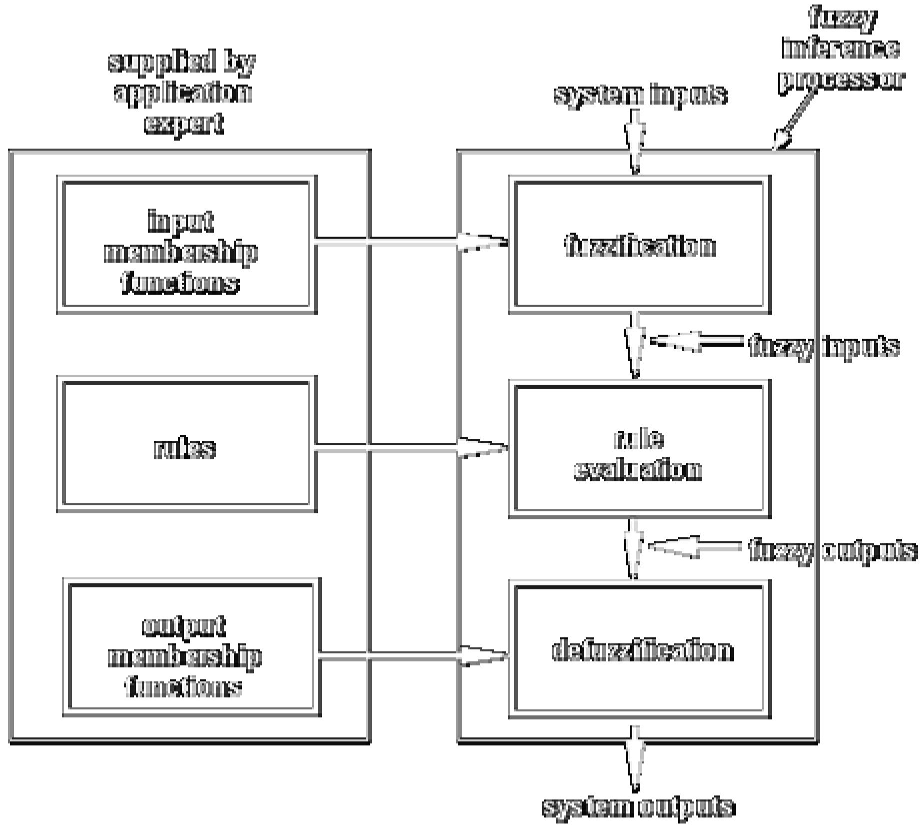


Fig. Fuzzy inferencing system

crisp inputs: distinct or exact inputs to a certain system variable, usually measured parameters external from the control system, e.g. 6 Volts.
label: descriptive name used to identify a membership function.
scope: or domain, the width of the membership function, the range of concepts, usually numbers, over which a membership function is mapped.
universe of discourse: range of all possible values, or concepts, applicable to a system variable.
When designing the number of membership functions for an input variable, labels must initially be determined for the membership functions. The number of labels correspond to the number of regions that the universe should be divided, such that each label describes a region of behavior. A scope must be assigned to each membership function that numerically identifies the range of input values that correspond to a label.
The shape of the membership function should be representative of the variable. However this shape is also restricted by the computing resources available. Complicated shapes require more complex descriptive equations or large lookup tables. The next figure shows examples of possible shapes for membership functions.

When considering the number of membership functions to exist within the universe of discourse, one must consider that:
i) too few membership functions for a given application will cause the response of the system to be too slow and fail to provide sufficient output control in time to recover from a small input change. This may also cause oscillation in the system.
ii) too many membership functions may cause rapid firing of different rule consequents for small changes in input, resulting in large output changes, which may cause instability in the system.
These membership functions should also be overlapped. No overlap reduces a system based on Boolean logic. Every input point on the universe of discourse should belong to the scopeof at least one but no more than two membership functions. No two membership functions should have the same point of maximum truth, (1). When two membership functions overlap, the sum of truths or grades for any point within the overlap should be less than or equal to 1. Overlap should not cross the point of maximal truth of either membership function.
The fuzzification process maps each crisp input on the universe of discourse, and its intersection with each membership function is transposed onto the μ axis as illustrated in the previous figure. These μ values are the degrees of truth for each crisp input and are associated with each label as fuzzy inputs. These fuzzy inputs are then passed on to the next step, Rule Evaluation.

Fuzzy If then Rules :

We briefly comment on so-called *fuzzy IF-THEN rules* introduced by Zadeh. They may be understood as partial imprecise knowledge on some crisp function and have (in the simplest case) the form IF x is A_i THEN y is B_i. They should **not** be immediately understood as implications; think of a *table* relating values of a (dependent) variable y to values of an (independent variable) x:

x	A_1	...	A_n
y	B_1	...	B_n

A_i, B_i may be crisp (concrete numbers) or fuzzy (small, medium, …) It may be understood in two, in general non-equivalent ways: (1) as a listing of n possibilities, called Mamdani's formula:

$$MAMD(x,y) \equiv \bigvee_{i=1}^{n} (A_i(x) \,\&\, B_i(y))$$

(where x is A_1 and y is B_1 *or* x is A_2 and y is B_2 or …). (2) as a conjunction of implications:

$$RULES(x,y) \equiv \bigwedge_{i=1}^{n} (A_i(x) \rightarrow B_i(y))$$

Both *MAMD* and *RULES* define a binary fuzzy relation (given the interpretation of A_i's, B_i's and truth functions of connectives). Now given a *fuzzy input* $A^*(x)$ one can consider the image B^* of $A^*(x)$ under this relation, i.e.,

$$B^*(y) \equiv \exists x(A(x) \ \& \ R(x,y)),$$

where $R(x,y)$ is $MAMD(x,y)$ (most frequent case) or $RULES(x,y)$. Thus one gets an operator assigning to each fuzzy input set A^* a corresponding fuzzy output B^*. Usually this is combined with some *fuzzifications* converting a crisp input x_0 to some fuzzy $A^*(x)$ (saying something as "x is similar to x_0") and a *defuzzification* converting the fuzzy image B^* to a crisp output y_0. Thus one gets a crisp function; its relation to the set of rules may be analyzed.

Rule Evaluation

Rule evaluation consists of a series of IF-Zadeh Operator-THEN rules. A decision structure to determine the rules require familiarity with the system and its desired operation. This knowledge often requires the assistance of interviewing operators and experts. For this thesis this involved getting information on tremor from medical practitioners in the field of rehabilitation medicine.

There is a strict syntax to these rules. This syntax is structured as:

IF antecedent 1 ZADEH OPERATOR antecedent 2 .... THEN consequent 1 ZADEH OPERATOR consequent 2..............

The antecedent consists of: input variable IS label, and is equal to its associated fuzzy input or truth value μ(x).

The consequent consists of: output variable IS label, its value depends on the Zadeh Operator which determines the type of inferencing used. There are three Zadeh Operators, AND, OR, and NOT. The label of the consequent is associated with its output membership function. The Zadeh Operator is limited to operating on two membership functions, as discussed in the fuzzification process. Zadeh Operators are similar to Boolean Operators such that:

AND represents the intersection or *minimum* between the two sets, expressed as:

$$\mu_{A \cap B} = \min[\mu_A(x), \mu_B(x)]$$

OR represents the union or *maximum* between the two sets, expressed as:

$$\mu_{A \cup B} = \max[\mu_A(x), \mu_B(x)]$$

NOT represents the opposite of the set, expressed as:

$$\overline{\mu_A} = [1 - \mu_A(x)]$$

The process for determining the result or rule strength of the rule may be done by taking the minimum fuzzy input of antecedent 1 AND antecedent 2, min. inferencing. This minimum result is equal to the consequent rule strength. If there are any consequents that are the same then the maximum rule strength between similar consequents is taken, referred to as maximum or max. inferencing, hence min./max. inferencing. This infers that the rule that is most true is taken. These rule strength values are referred to as fuzzy outputs.

Defuzzification

Defuzzification involves the process of transposing the fuzzy outputs to crisp outputs. There are a variety of methods to achieve this, however this discussion is limited to the process used in this thesis design.

A method of averaging is utilized here, and is known as the Center of Gravity method or COG, it is a method of calculating centroids of sets. The output membership functions to which the fuzzy outputs are transposed are restricted to being singletons. This is so to limit the degree of calculation intensity in the microcontroller. The fuzzy outputs are transposed to their membership functions similarly as in fuzzification. With COG the singleton values of outputs are calculated using a weighted average, illustrated in the next figure. The crisp output is the result and is passed out of the fuzzy inferencing system for processing elsewhere.

Fuzzy Rule base and Approximate Reasoning an example:

What is fuzzy linguistic variable? Algebraic variables take numbers as values, while linguistic variables take words or sentences as values.

For example, let x be a linguistic variable with a label „temperature‟. The universe ofdiscourse is temperature. In that universe, I am looking at a fuzzy variable x when I describe the temperature. The fuzzy set temperature denoted as T can be written as T = very cold,cold, normal, hot or very hot.

For each linguistic value, we get a specific membership function.

These are necessary because in the traditional sense, when we express worldly knowledge, we express them in natural language. So here it is. From computational perspective, such worldly knowledge can be expressed in terms of rule base systems.

Rule based systems:

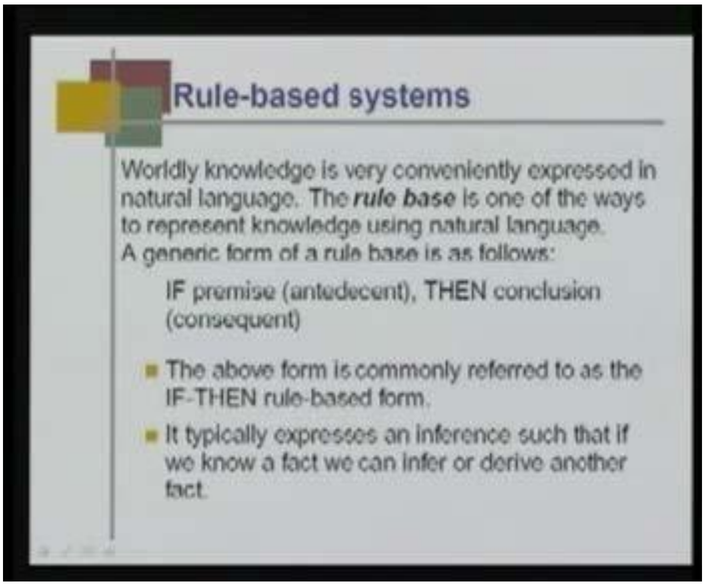


Fig. Basics of rule based system

The above form is commonly referred to as the IF-THEN rule-based form. It typically expresses an inference such that if we know a fact, we can infer or derive another fact. Given a rule, I can derive another rule or given a rule, if I know a rule and the associated relation, then given another rule, I can predict what should be the consequence.

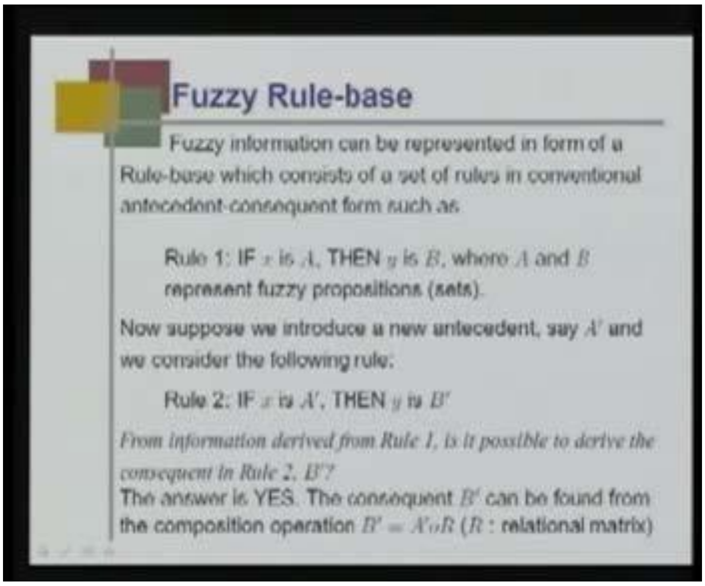


Fig.Fuzzy rules

This is a fuzzy rule base. Any worldly knowledge can be expressed in form in the form of a rule base. Now, when I talk about fuzzy rule base, fuzzy information can be represented in the form of a rule base, which consists of a set of rules in conventional antecedent and consequent form such as if x is A, then y is B, where A and B represent fuzzy propositions (sets). Suppose we introduce a new antecedent say A dash and we consider the following ruleif x is A dash, then y is B dash, from the information derived from rule 1, is it possible to derive the consequent in rule 2, which is B dash?

The consequent B dash in rule 2 can be found from composition operation B dash equal to A dash. This is called the compositional rule of inference, the compositional operator with R.

Fuzzy implication Relation:

A fuzzy implication relation is another category, which will call Zadeh implication. This is if p implies q may imply either p and q are true or p is false. What we are saying is that just like a local Mamdani rule, we say p and q are true imply either p and q are true or p is false. Thus, p implies q means…. p and q are simultaneously true, which is Mamdani local rule or if p is false, then p implies q has no meaning or p is false. This has taken an extra logic that is p and q or not p.

Thus, the relational matrix can be computed as follows. If I look at this, what is p and q? p and q means minimum of mu$_A$ x and mu$_B$ y. What is not p? 1 minus μ_A x. This entire thing has to be maximum of minimum of these and this, which is this statement.µ, the relational matrix elements are computed using this particular expression. Given a set of rules, we just learnt various schemes by which we can construct a relational matrix between the antecedent and the consequent. The next step would be to utilize this relational matrix for inference. This method is commonly known as compositional rule of inference, that is, associated with each rule we have a relational matrix. So, given a rule means given a relational matrix and given another antecedent, we compute a consequent.

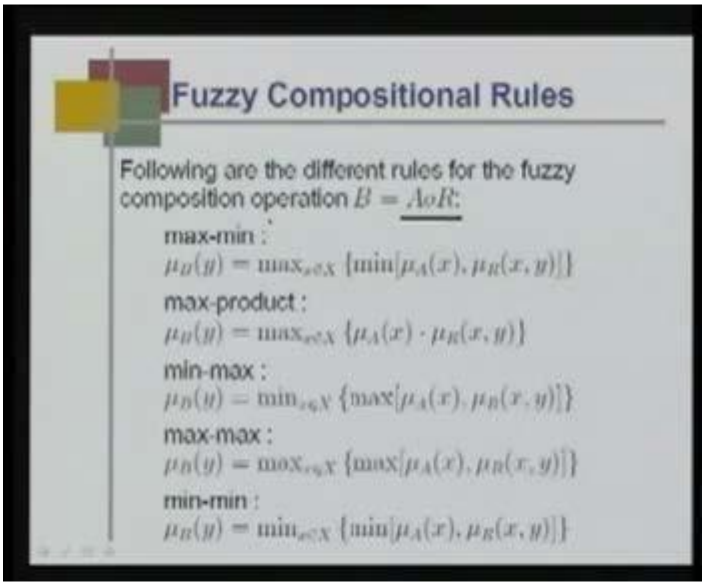


Fig. Compositional rules

This is derived using fuzzy compositional rules. The following are the different rules for fuzzy composition operation, that is, B equal to A composition R. R is the relational matrix associated with a specific rule, A is a new antecedent that is known, R is known, B is the newconsequent for the new antecedent A. I have to find out what is B for this new A, given R. That is computed by A composition R and we have already discussed in the relation class thatthere are various methods and max-min is very popular.

First, we compute min and then max. Similarly, max-product: instead of min, we take the product and compute what is the maximum value. Similarly, min-max: instead of max-min, it is min-max. First, max and then min. Next, max-max and min-min. One can employ these looking at the behavior of a specific data.

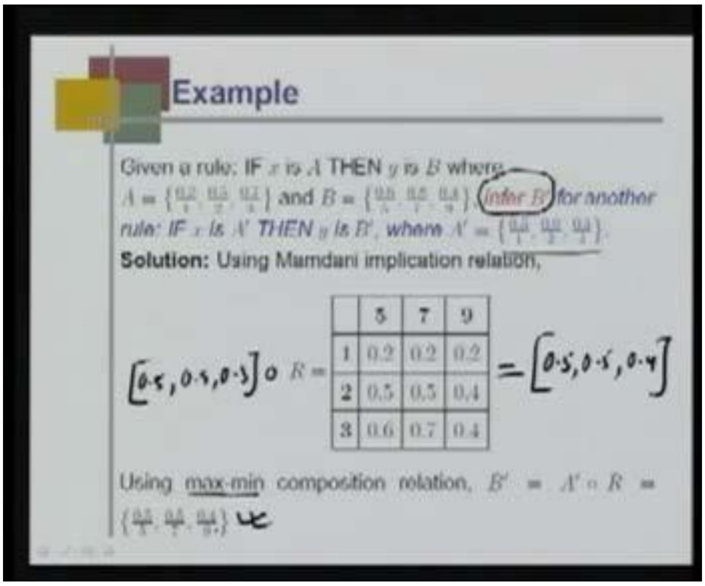


Fig. Example of Compositional rules

Now, we will take an example.
We are given a rule if x is A, then y is B, where A is this fuzzy set: 0.2 for 1, 0.5 for 2, and 0.7 for 3. This is a discrete fuzzy set. B is another fuzzy set that defines fuzzy membership

0.6 for 5, 0.8 for 7, and 0.4 for 9. The question is infer B dash for another rule if x is A dash, then y is B dash, where A dash is known. A is known, B is known, and A dash is known. What we have to find out is what B dash is. Infer B dash is the question that is being asked Using Mamdani implication relation, first we will find out between A… the first rule, that is, if x = A, then y is B. The relational matrix associated with this rule is…. For R, how do we compute? A elements are 1, 2, and 3 and B elements are 5, 7, and 9. We have to find out now for 0.2. Here, we compare with all the elements in point B and with each element, we found what the minimum is. The minimum is always 0.2. Hence, the maximum of that is always 0.2. I have to find out the relational matrix between A and B.

The Mamdani principle means minimum, so between 1 and 5, 1 is associated with 0.2, and 5 is associated with 0.6, so the minimum is 0.2. Similarly, 1 is associated with 0.2, 7 is associated with 0.8, so for 1 and 7, the minimum is 0.2. Similarly, 1 is associated with 0.2, 9is associated with 0.4, so from 1 to 9, the minimum membership is 0.2. Similarly, you can seethat from 2 to all the elements 5, 7, 9, the minimum are 0.5, 0.5, and 0.4. Similarly, from 3 to 5, 7, and 9, we have 0.6, 0.7, and 0.4. These are the minimum fuzzy memberships between an element in A to element in B. That is how we compute the relational matrix.

Once we compute the relational matrix, then we use max-min composition relation to find out what is B dash, which is A dash (which is 0.5, 0.9, and 0.3) composition R and you can compute. This is my R. I have to find out my matrix. This is 0.5, 0.9, and 0.3. So this composition R is… you can easily see I take this row vector, put along the column matrix and I see what is the minimum for each case. You can easily see 0.2 will be minimum here, will be minimum here, 0.3 and maximum is 0.5.

The first element is 0.5. Again, I take this place in parallel with this column and then, I find first minimum here is 0.2, here 0.5, here 0.3 and then maximum is again 0.5. Again, I take the same row vector, put along this column vector and then, I find here the minimum is 0.2, here minimum is 0.4, here minimum is 0.3 and the maximum is 0.4. This is the relation, this is the answer. This is our B dash. Given A, this is my B dash using fuzzy compositional principle or relation.

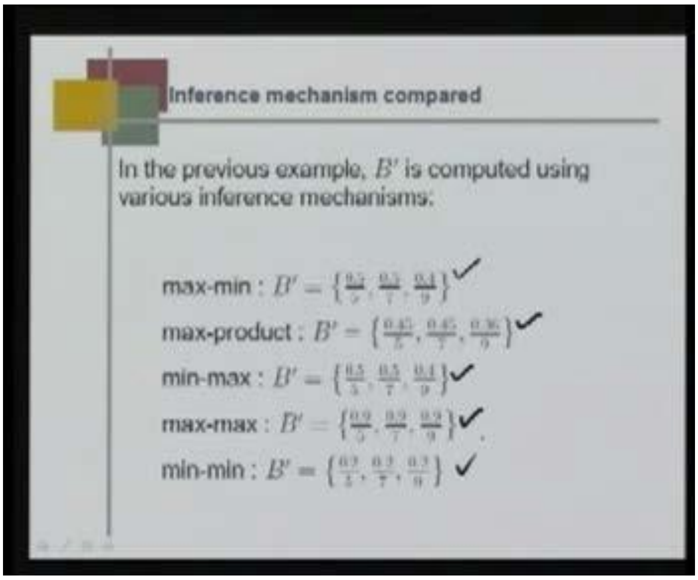


Fig. Comparison of compositional rules

There are other mechanisms also that we discussed. For the same example, if you use max-min, you get B dash; for max-product, you get another B dash; for min-max, you get another. Min-max and max are same for this example. Then, for max-max, you see that all the fuzzy membership are the maximum values and for min-min, they are the minimum values here.

Approximate reasoning:
means given any logical system, we do not have, it is very difficult to make an exact result. That is why from engineering perspective, we are more liberal. We do not want to be so precise. As long as our system works, we are happy; if our control system works, we are happy.

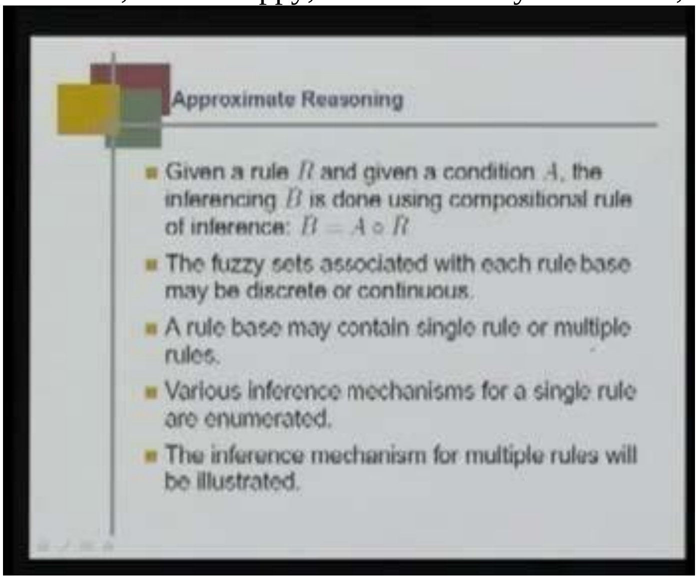


Fig. Approximate reasoning

Approximate reasoning. We have set up rules so we use a specific compositional rule of inference and then we infer the knowledge or the consequence. Given a rule R (R is the relational matrix associated with a specific rule) and given a condition A, the inferencing B is done using compositional rule of inference B equal to A composition R. The fuzzy sets associated with each rule base may be discrete or continuous, that is, A may be discrete or A and B may be discrete or continuous.
A rule base may contain a single rule or multiple rules. If it is continuous, I cannot define what the R relational matrix is. It is very difficult because it will have infinite values. R is notdefined. That is why for continuous, we apply compositional rule of inference but the methodto compute is different. A rule base may contain single rule or multiple rules. Various inference mechanisms for a single rule are enumerated. Various mechanism means we talked about min-max, max-min, max-max, min-min and so on. The inference mechanism formultiple rules.
Single rule:
Now, we will take the examples one by one. Single rule with discrete fuzzy set. We talked about a fuzzy set that may consist of a single rule or multiple rules. It can be discrete fuzzy set or a continuous fuzzy set. We will try to understand how to make approximate reasoning for such a rule base using the methods that we just enumerated. For each rule, we compute what is the relational matrix if it is discrete fuzzy set and then we use compositional rule of inference to compute the consequence given an antecedent. That is for discrete fuzzy set. We have already talked about this but again, for your understanding, I am presenting another example for single rule with discrete fuzzy set.

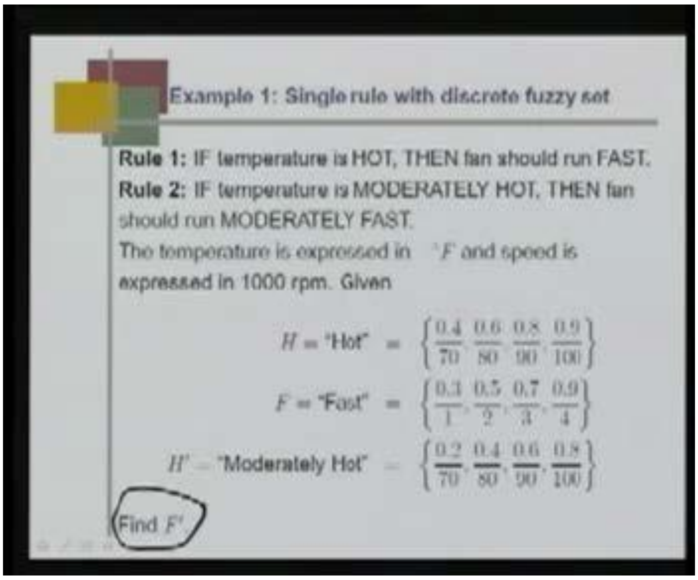


Fig. Single rule

Rule 1: If temperature is hot, then the fan should run fast. If temperature is moderately hot, then the fan should run moderately fast. In this example, we are given the temperature is in degree Fahrenheit and the speed is expressed as 1000 rpm. The fuzzy set for hot H is for 70 degree Fahrenheit, 80 degree Fahrenheit, 90 degree Fahrenheit, and 100 degree Fahrenheit, the membership values are 0.4, 0.6, 0.8, and 0.9. Similarly, for the fuzzy set F, for which the fan should run fast, the fuzzy set is for 1000 rpm, the membership is 0.3, for 2000 rpm, the membership is 0.5, for 3000 rpm, the membership 0.7, and for 4000 rpm, the membership is 0.9.

Given H dash, which is moderately hot, to be for 70… moderately hot means it is a little more hot. So, same temperature obviously and their corresponding membership values will reduce, because if I am describing moderately hot, they will have the same temperature but the membership values will be less. You can easily see here that for 70, instead of 0.4, now it is 0.2; for 80, instead of 0.6, it is 0.4; for 90, instead of 0.8, it is 0.6; for 100, instead of 0.9, it is 0.8. This is moderately hot. Now, the question is find F dash.

I hope you are clear with this question. The question is very simple. We are given rule 1, we have defined what is the fuzzy set hot and fuzzy set fast by these two statements and in the second rule for moderately hot, we know the fuzzy set. We do not know what the fuzzy set is corresponding to moderately hot, that is, moderately fast. We do not know moderately fast. Find out F dash. If H, then F. If H dash, then F dash. Find out F dash. First, what do we do?

Corresponding to rule 1, we found out what is R. This is for rule 1. We knew that the membership functions for H were 0.4, 0.6, 0.8, and 0.9, and for fast, the membershipfunctions where 0.3, 0.5, 0.7, and 0.9. If you look at this, these are my H values, the crisp values: 70 degree Fahrenheit, 80 degree Fahrenheit, 90 degree Fahrenheit, and 100 degree Fahrenheit. This is my speed: 1000 rpm, 2000 rpm, 3000 rpm, and 4000 rpm.

Between 70 and 1000 rpm, the entry would be minimum of these two (Refer Slide Time: 41:57), which is 0.3. Similarly, between 0.4 and 0.5, the minimum would be again 0.4 and then between 0.4 and 0.7, it will be 0.4, and for 0.4 and 0.9, it is 0.4.

Similarly, we go to the next one, which is 0.6. For 0.6, 0.3 minimum 0.3, for 0.6 and 0.5, the minimum is 0.5, for 0.6 and 0.7, minimum is 0.6, for 0.6 and 0.9, it is 0.6. Similarly, you can fill all other cells here with their values: 0.3, 0.5, 0.7, 0.8, 0.3, 0.5, 0.7, and 0.9. This is my relation matrix associated with rule 1: if H, then F. Now, what I have to do is I have to find out F dash given H dash, using the fuzzy compositional rule of inference, which is represented like this.

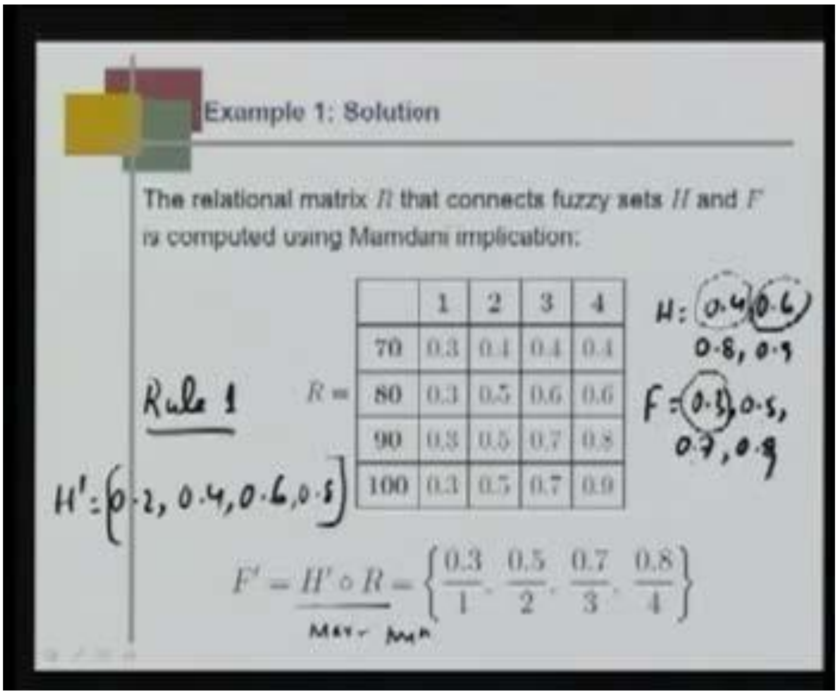


Fig. Relational matrix

F dash is H dash compositional rule of inference with R. This is max-min composition operation. First, we take the min and then compute. H dash is given as 0.2, 0.4, 0.6, and 0.8.

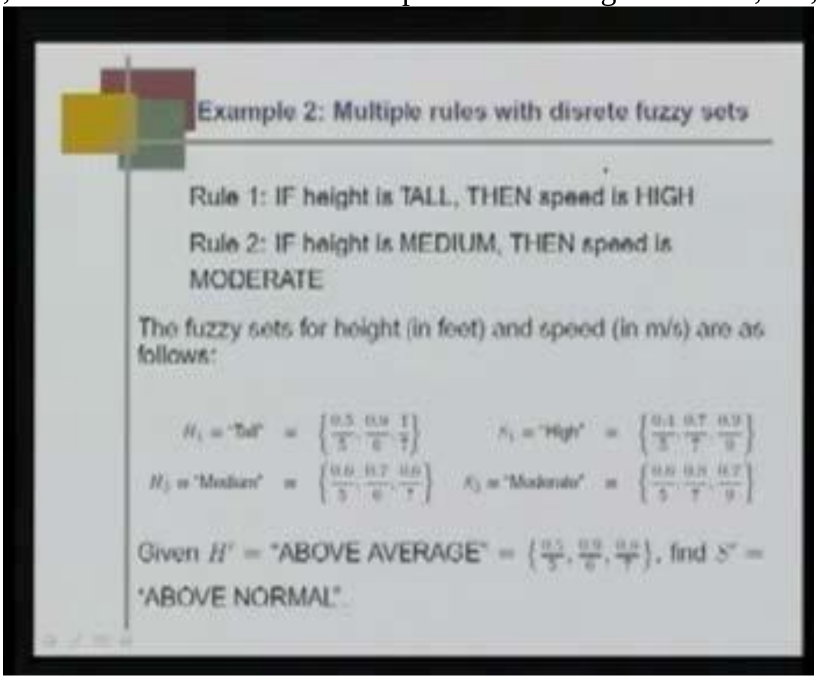


Fig. Multiple rules

This is my H dash (moderately hot) and I have to do compositional inference between H dash and R. Again, I am repeating so that you understand how to compute it. You put this row

vector along this column vector first . For each element, you find out what is the minimum. You see that here it is 0.2, 0.3, 0.3, and 0.3 and the maximum of that is 0.3.
Similarly, you take again these values and put them here vertically. Here, the minimum is 0.2, here 0.4, here 0.5, here 0.5, and maximum is 0.5. I am sure you will see here it is 0.7, but in this case, you find that if you take this here, it is 0.2, here 0.4, here 0.6, here 0.8, and maximum is 0.8. F dash is 0.3, 0.5, 0.7, and 0.8. That is how we infer or we do approximate reasoning for a rule base. This is a very simple case.

Multiple rule:

There are two rules now. Rule 1 is if height is tall, then speed is high. Rule 2: if height is medium, then speed is moderate. This is describing a rule for a person as to how fast he can walk. Normally, those who are tall can walk very fast and those who are short, naturally their speed will be less. This is one fuzzy rule that expresses the speed of a person while walking. If height is tall, then speed is high and if height is medium, then speed is moderate. For this, the fuzzy memberships are defined as tall, high, medium, and moderate.
Tall is 0.5, 0.8, and 1 for various feet like 5, 6, and 7. For speed is high, for 5 meter per second, 7 meter per second, and 9 meter per second, the corresponding membership valuesare 0.4, 0.7, and 0.9. For H2, which is medium height, the corresponding fuzzy membership… you can easily see that when I say medium in this fuzzy set, 5 has 0.6, 6 has 0.7, and 7 has 0.6. The moderate speed is 0.6 for 5 meter per second, 0.8 for 7 meter per second, and 0.7 for 9 meter per second. If this is the fuzzy set given, now the question is given H dash, which is above average, and the corresponding fuzzy set is 0.5, 0.9, 0.8 for three different heights, find S dash, the speed above normal. I hope the question is very clear to you.

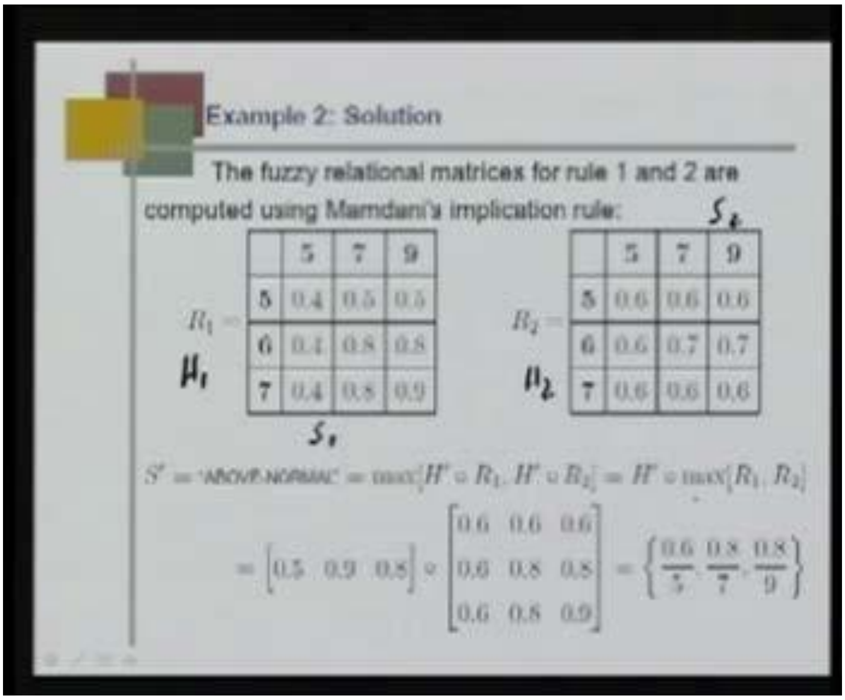


Fig. Relational matrix for 2 rules

We have two rules. If height is tall, then speed is high; tall is defined and high is defined. If height is medium, then speed is moderate. I have already defined the fuzzy sets for both medium as well as moderate. They are all discrete fuzzy sets. Now, you are presented with new data and what is that new data? You are presented with a data called above average, which is 0.5, 0.9, and 0.8 for three different heights for 5, 6, and 7. Then, find S dash equal toabove normal, that is, if height is above average, then the speed should be above normal.

This is the solution of this example. We have two rules. Naturally, we will have tworelational matrices: R1 for rule 1 and R2 for rule 2. I will not go in detail of how we compute.You simply you go the antecedent and consequent, look at the membership function, find the minimum for each entry. Here, these are the heights and these are the speeds; 5, 6, 7 feet isthe height and 5, 7, and 9 meter per second are the speeds of the individuals.
Now, you check the fuzzy sets and corresponding to each fuzzy set, find out what is the minimum membership function. For 5, 5, you will find the membership function is 0.4, minimum 0.5, 0.5, 0.4, 0.8, 0.8, 0.4, 0.8, 0.9. You can verify this. Similarly, R2 can be found out. Taking the minimum membership entry between these two fuzzy sets, that is,
if I say this is H1 and S1 and this is H2 and S2. Look at these two fuzzy sets, find out what the minimum entries are for each relation and then, how do we compute S dash above normal? We have now two relational matrices. It is very simple. We do two composition operations: H dash composition with R1 (this one) and again, H dash composition R2 and then, we take the maximum of that, maximum of these two.

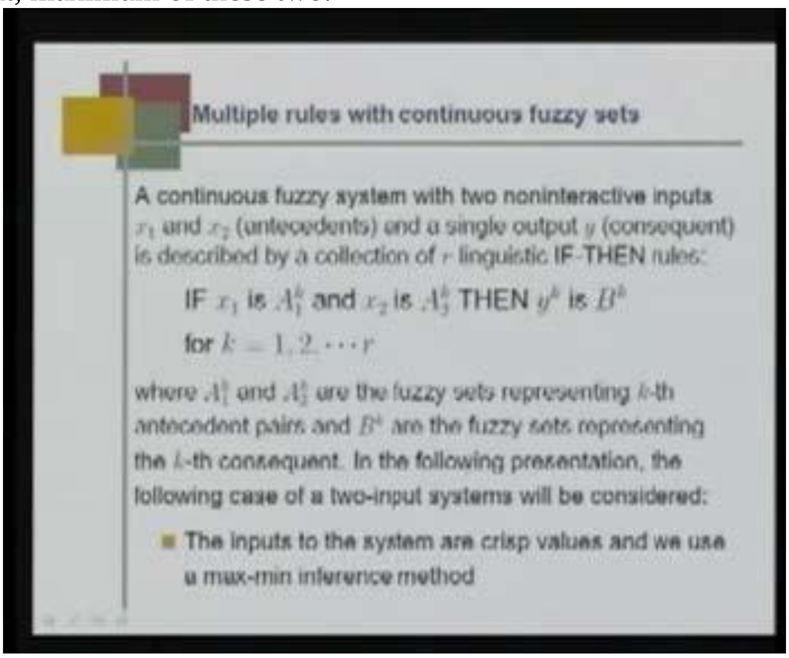


Fig. Multiple rule with continuous fuzzy sets

You can easily see that the maximum of H dash composition R1, H dash composition R2. You can easily see that because H dash is common, this particular expression is the same asH dash composition max of R1 and R2. This is R1 and R2. We look at all those entries wherever it is the maximum: for 0.4 and 0.6, the maximum is 0.6; for 0.5 and 0.6, the maximum is 0.6; for 0.5 and 0.6, the maximum is 0.6. You see the last element here 0.9 here and 0.6, so this is 0.9. Like that, for all entries of R1 and R2, whatever the maximum values, you put these values here (that is called maximum R1 and R2) and take a composition with Hdash. So H dash composition max of R1 and R2. H dash is already given as 0.5, 0.9, and 0.8. If you do this composition, you get 0.6, 0.8, and 0.8. I hope this clears your concept of how we compute or we do approximate reasoning in a rule base. Similarly, if there are multiple rules, we have no problem and we can go ahead with the same principle.

The last section is the multiple rules with continuous fuzzy sets. We talked about discrete fuzzy set, but if it is continuous fuzzy sets, how do we deal with that? Normally, a continuousfuzzy system with two non-interactive inputs x1 and x2, which are antecedents, and a single output y, the consequent, is described by a collection of r linguistic IF-THEN rules Where therule looks like this: If x1 is A1 k and x2 is A2 k, then y k is B k, where k is 1, 2 up to r. Thisis the k th rule. Similarly, we can have rule 1, rule 2, rule 3, up to rule r. In this particularrule, A1 k and A2 k are the fuzzy sets representing the k th antecedent pairs and B k are the fuzzy sets representing the k th consequent. In the following presentation, what we will do now is we will take a two-input system and two-rule system just to illustrate how we infer from a rule base where the fuzzy sets are continuous. The inputs to the system are crisp values and we use a max-min inference method.

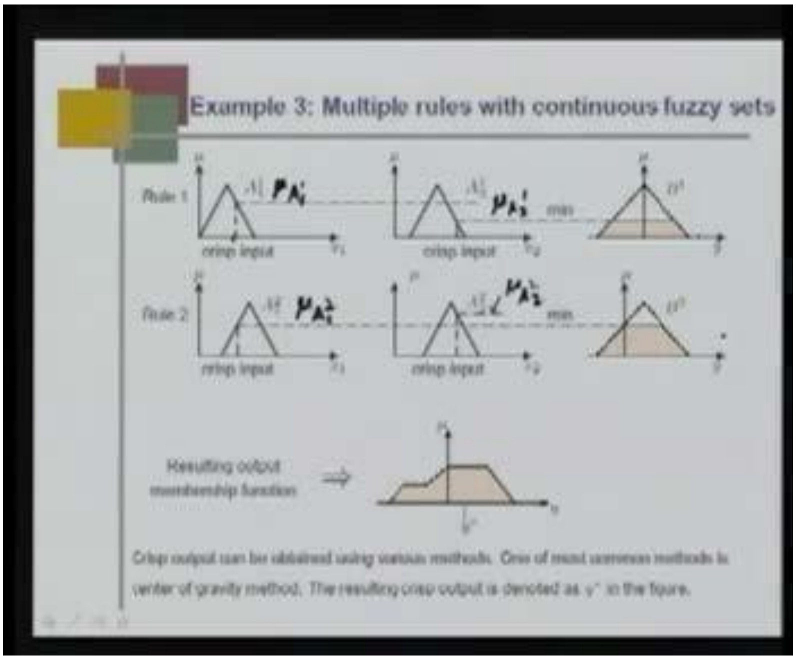


Fig. Viewing multiple rules

We have two rules here represented graphically. You can see there are two variables x1 and x2. There are two fuzzy variables and for each rule, we have a consequent y. The first rule says that if x1 is A1 1 and x2 is A2 1, then y is B1.

Similarly, if x1 is A1 2, x2 is A2 2, then y is B2. Now, how do we infer? Given a crisp input, a new input is given, crisp input in the domain of x1 and another crisp input in the domain of x2. There can be a system whose two variables can be temperature as well as pressure. You can easily think x1 to be the temperature and x2 to be the pressure. For example, for a particular given system, you found out the temperature to be 50 degrees centigrade and pressure to be some value. Given these two quantities, crisp quantities, how do we infer what should be y?

The crisp input is given – temperature. Now, you find out corresponding membership values here. Corresponding to this crisp input, we get the membership value in rule 1 as µA1 1 and for the same crisp input, this rule 2 will provide you muA1 2. Now, in the second fuzzy variable, given crisp input, rule 1 will compute µA2 1 and for the second one, the second rule, the same crisp input would give this one, which is muA2 2. Once we find out these

membership values, what do we do? We graphically see which is minimum between µA1 1 and µA2 1. The minimum is µA2 1. We take that and we shade these areas in consequence. Now, we take the second rule. We find between µA1 2 and µA2 2, the minimum is µA1 2. We take that minimum and shade the area and consequent part of this rule 2. Now graphically, we add these two taking the maximum. First, min and then max. You can easily see that when I overlap this figure over this figure, I get this particular figure. You overlap this second figure on the first figure or first figure on the second figure and take the resultant shaded area. After taking this resultant shaded area…. Once you find this shaded area, the next part is to see what is y given a crisp value. There are many methods, but we will focus inthis class or in this course on only one method, that is, center of gravity method(COG).
Obviously, if I take this figure and find out what is the center of gravity, it is this value y star. The crisp output can be obtained using various methods. One of the most common method is the center of gravity method. The resulting crisp output is denoted as y star in the figure. This is y star. What we learnt in this is given a crisp input 1 and crisp input 2 and given two fuzzy rules, how do we infer correspondingly a crisp output? Our data is crisp, but we are doing fuzzy computation. Hence, rules are fuzzy. We take this data to the fuzzy rule base and then fuzzify them through fuzzification process. Graphically, we find what is the net shaded area using the max principle. We found out the shaded area for each rule in consequent taking the min principle. Taking the max principle, we found out the resultant area and then, y star is thecenter of gravity of these areas.

LECTURE-8

Fuzzy Control system or Fuzzy Inference System:

Categories:

1. Mamdani type and
2. Takagi–Sugeno type (T-S or TSK for short form T. Takagi, M. Sugeno, and K. T. Kang).

Mamdani type fuzzy systems:

These employ fuzzy sets in the consequent part of the rules. This is a Mamdani type fuzzy logic controller. What they do is that the consequent part itself takes the control action; the incremental control action is described in the consequent part of each rule.

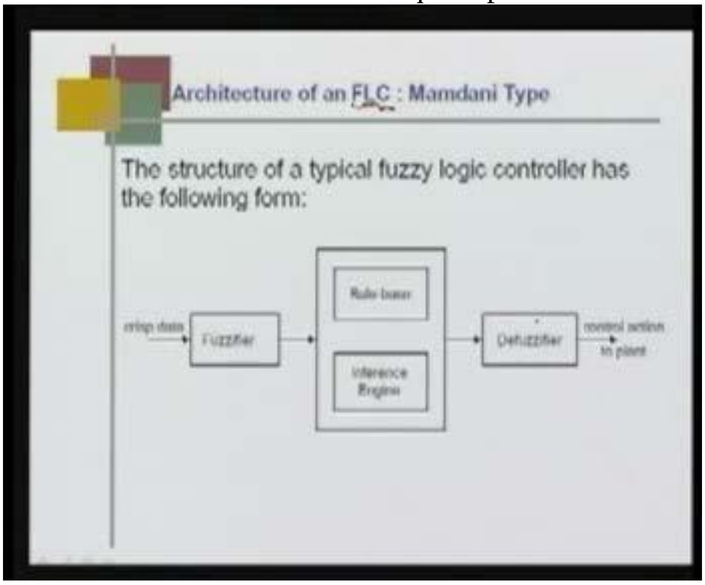


Fig. Architecture of FLC

The actual data that the controller is receiving is crisp data or classical data that has a definite value. That crisp data goes to the fuzzy logic controller and it has these four components that you can see: fuzzifier, rule base, inference engine and defuzzifier.

Fuzzifier. In a fuzzy logic controller, the computation is through linguistic values, not through exact computation. Naturally, the fuzzifier would fuzzify the crisp data. In case of temperature, I can say it is hot, medium-hot, cold, medium-cold, very hot and normal. These are the fuzzifier. That means given a crisp data or the value of temperature say 40 degrees, then I have to now convert to various linguistic values and each linguistic value will be associated with a specific membership function. That is fuzzifier.

Once the data has been fuzzified, then it goes to the rule base and using an inference mechanism…. The inference is taking place in fuzzy term, not in classical term and after a fuzzy inference takes place about the decision or about the control action, we place a defuzzifier. What this defuzzifier does is it converts the fuzzy control action to a crisp control action.

In general, what we can say is the principal design parameters of a fuzzy logic controller are the following: fuzzification strategies and interpretation of a fuzzification operator. How do we fuzzify a crisp data? In the database, the discretization or normalization of universe of discourse is done, because we must know the range of data one will encounter in an actual plant. Accordingly, the normalization must be done so that we are taking into account all possible values of data that one may encounter in a physical plant.

Fuzzy partition of the input and output spaces:

If I know the dynamic range of an input to the controller and the input to the plant (input to

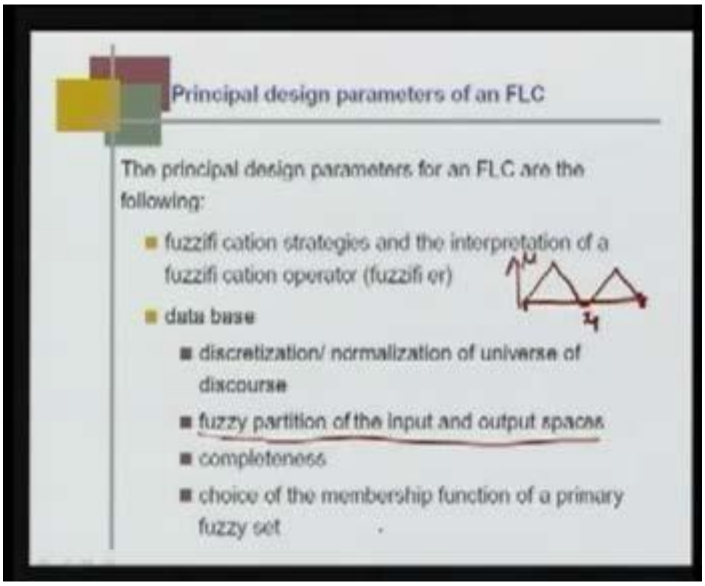


Fig. Parameters to be designed in FLC

the plant is actually output to the controller)… if I know the dynamic range, then in that dynamic range, I must learn how to do fuzzy partition of the input and output space and this fuzzification suits such the process should be complete in the sense.… You see that I am drawing a universe of discourse here. This is the real value for a specific variable x_1. If I have defined a fuzzy set like this and like this, you can easily see that this part of the data is not associated with any fuzzy membership. This is μ and this is x_1 and unfortunately, this part is not associated with any membership.

This fuzzification process is not complete. That means the entire universe of discourse in a specific domain, wherever there are control systems…. There are various kinds of control systems: process control, robot control, and aircraft control. Every control system is associated with some input data and some output data. All possible input data and all possibleoutput data should be associated with a specific linguistic value as well as a membership function.

Rule base:

Once fuzzification is done, how do we create a rule base? As I said, typically, in the rule base, the two variables that are most important are error and change in error and we also showed why it is so. Rule base. Choice of process state input variables and control variables. You know that if I am implementing a fuzzy state feedback controller, then, a fuzzy state feedback controller u would be minus K x. So, x is the states of the system, whereas if I am implementing a fuzzy PID controller, then it will be u old plus K delta u k. Here, this delta uk is a function of error and change in
error, whereas, in a state feedback controller, this is a common signal r and so, the control action is dependent on state x1, x2, and xn.

Source and derivation of fuzzy control rules.

How do I derive these rules? What is the basis? Types of fuzzy control rules. A type of fuzzy control rule means whether it is a PID controller, fuzzy PID controller or it is a fuzzy state feedback controller. Similarly, completeness of fuzzy control rules means given any crisp data in the domain of input space as well as output space, do I have in my rule base a specific rule associated with this data? If I do not have any rule for this data, then the FLC will fail. That is meaning of completeness of fuzzy control rules.

Fuzzy inference mechanism:

We have already talked about what is fuzzy inference mechanism. Given multiple rules, how do we infer the consequence part? Defuzzification strategies and the interpretation of fuzzification operator. Once the fuzzy inference is done, from the fuzzy inference, how do I get a crisp value or a crisp control action? This is called defuzzification.

This is how we fuzzify a crisp data to fuzzy data or we make them fuzzy, that is, the crisp input for variable x and x …. Actually, this is not x and x but e and delta e are converted to fuzzy sets using tr[1]iangul[2]ar membership functions.[1] It is [2]not always triangular, it can be anything, but normally in control literature, most of these membership functions are triangular functions.

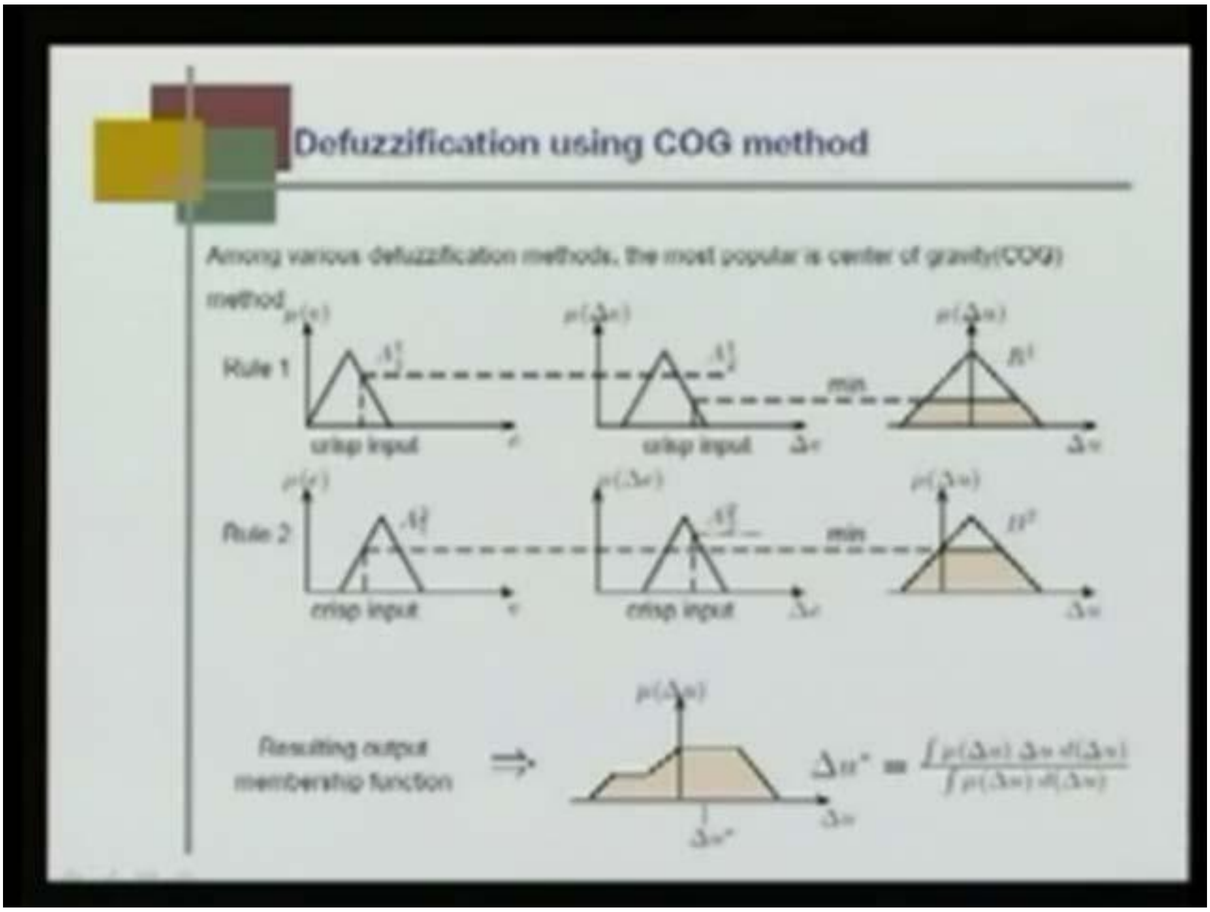


Fig. Defuzzification

Defuzzification. Once I know how to do the fuzzification, defuzzification is explained in the following diagram. You see that among various defuzzification methods, the most popular is center of gravity method. How do I do it? Crisp input is given at any situation, any k th sampling instant and the fuzzy logic controller gets the command signal, gets the actual output of the plant, computes the error, computes the change in error and then, those crisp values are fed into the fuzzification layer. Then, you have the membership function. You passon those fuzzy data to the rule base and then, for a specific rule base… You see that in rule 1, you see that if you compare the membership μ A_1 1 and μ A_2 1, μ A_2 1 is the minimum and correspondingly, you shade the zone of action. This is delta u. How much should be the incremental control action? This is my shaded portion or shaded portion of my control action.

Now I take a second one, second rule and there again, I evaluate the fuzzy membership function A_1 2 and A_2 2. You see that the membership function in A_1 2 is less. Corresponding

to that, we shade the incremental control action. Now, you see that if I take the maximum of these two shaded zones, I get this (Refer Slide Time: 38:03), maximum of this. After I get, this is the fuzzy decision, this is the fuzzy incremental control action, but how do I convert this fuzzy incremental control action to a crisp action? That is by the center of gravity method. In the center of gravity method, I integrate µ delta u d delta u upon integration of µ d delta u. If I integrate this function, I get somewhere here to be the center of gravity. delta µ star is this value, which is graphically shown here. We discussed about Mamdani type fuzzy logic controller.

LECTURE-9

Takagi–Sugeno fuzzy systems:

The number of rules required by the Mamdani model are reduced here. They employfunction of the input fuzzy linguistic variable as the consequent of the rules. What happens here is that a fuzzy dynamic model is expressed in the form of local rules. Local rule means if there are two variables x1 = a and x2 = b, then the plant dynamics can be represented either as a linear dynamical system or a nonlinear dynamical system, as a known dynamical system.

TSK Fuzzy Rule

• If x is A and y is B then z = f(x,y)

– Where A and B are fuzzy sets in the antecedent, and

– Z = f(x,y) is a crisp function in the consequence.

• Usually f(x,y) is a polynomial in the input variables x and y, but it can be any function describe the output of the model within the fuzzy region specified by the antecedence of the rule.

First order TSK Fuzzy Model

• f(x,y) is a first order polynomial

Example: a two-input one-output TSK

IF x is Aj and y is Bk then zi= px+qy+r

The degree the input matches ith rule is typically computed using min operator: wi = min(µAj(x), µBk(y))

• Each rule has a crisp output

• Overall output is obtained via weighted average (reduce computation time of defuzzification required in a Mamdani model)

$z = \sum_i w_i z_i / \sum_i w_i$

To further reduce computation, weighted sum may be used, I.e.

$z = \sum_i w_i z_i$

Example #1: Single-input

• A single-input TSK fuzzy model can be expressed as

– If X is small then Y = 0.1 X +6.4.

– If X is medium then Y = -0.5X +4.

– If X is large then Y = X-2.

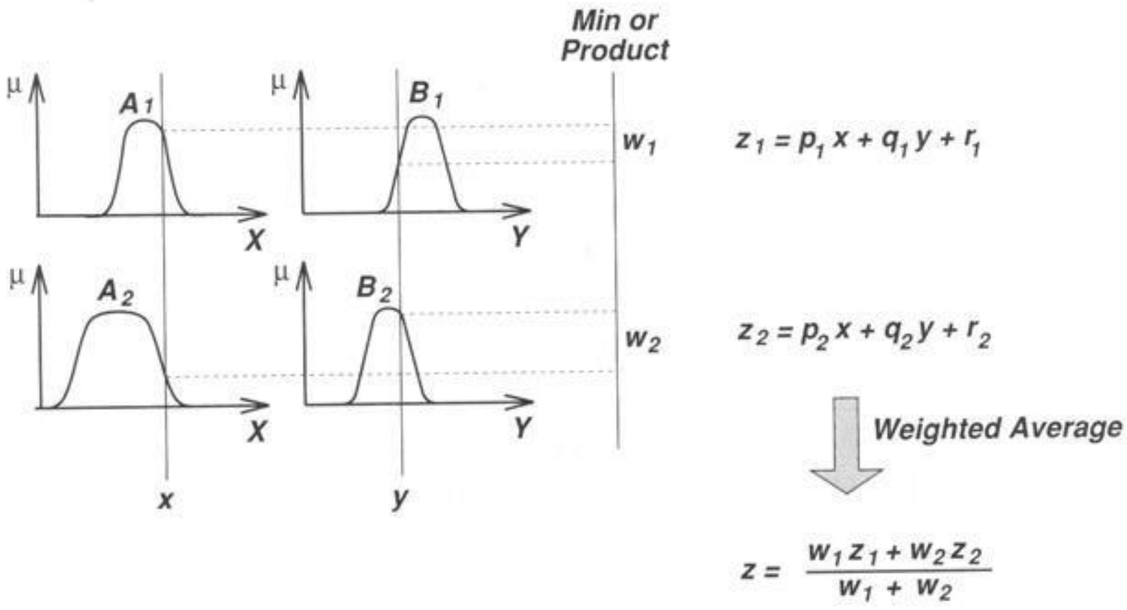


Fig. Rules in first order TSK fuzzy model

<u>Example #2 : Two-input</u>

• A two-input TSK fuzzy model with 4 rules can be expressed as

– If X is small and Y is small then Z = -X +Y +1.

– If X is small and Y is large then Z = -Y +3.

– If X is large and Y is small then Z = -X+3.

– If X is large and Y is large then Z = X+Y+2.

<u>Zero-order TSK Fuzzy Model</u>

• When f is constant, we have a zero-order TSK fuzzy model (a special case of the Mamdani fuzzy inference system which each rule‟s consequent is specified by a fuzzy singleton or a pre defuzzified consequent)

• Minimum computation time Overall output via either weighted average or weighted sum is always crisp

• Without the time-consuming defuzzification operation, the TSK (Sugeno) fuzzy model is by far the most popular candidate for sample data-based fuzzy modeling.

A general Takagi–Sugeno model of N rules for any physical plant, a general T–S model of N rules is given by Rule_i. This is the i th rule. If x_1 k is a specific fuzzy set M_1 i and x_2 k is another specific fuzzy set M_2 i and so on until x_n k is another fuzzy set M_n i, then the system dynamics locally is described as x k plus 1 is A_i x k plus B_i u k, where i equal to 1, 2 until N, because there are N rules.

Advantages over Mamdani model:

1. Less computation
2. Less time consuming
3. Simple
4. Mostly used for sample data based fuzzy modelling

Tsukamoto Fuzzy Models:

• The consequent of each fuzzy if-then rule is represented by a fuzzy set with monotonical MF

– As a result, the inferred output of each rule is defined as a crisp value induced by the rules‟ firing strength.

• The overall output is taken as the weighted average of each rule‟s output.

Example: Single-input Tsukamoto fuzzy model

• A single-input Tsukamoto fuzzy model can be expresses as

– If X is small then Y is C1

– If X is medium then Y is C2

– If X is large then Y is C3

Example: Single-input Tsukamoto fuzzy model

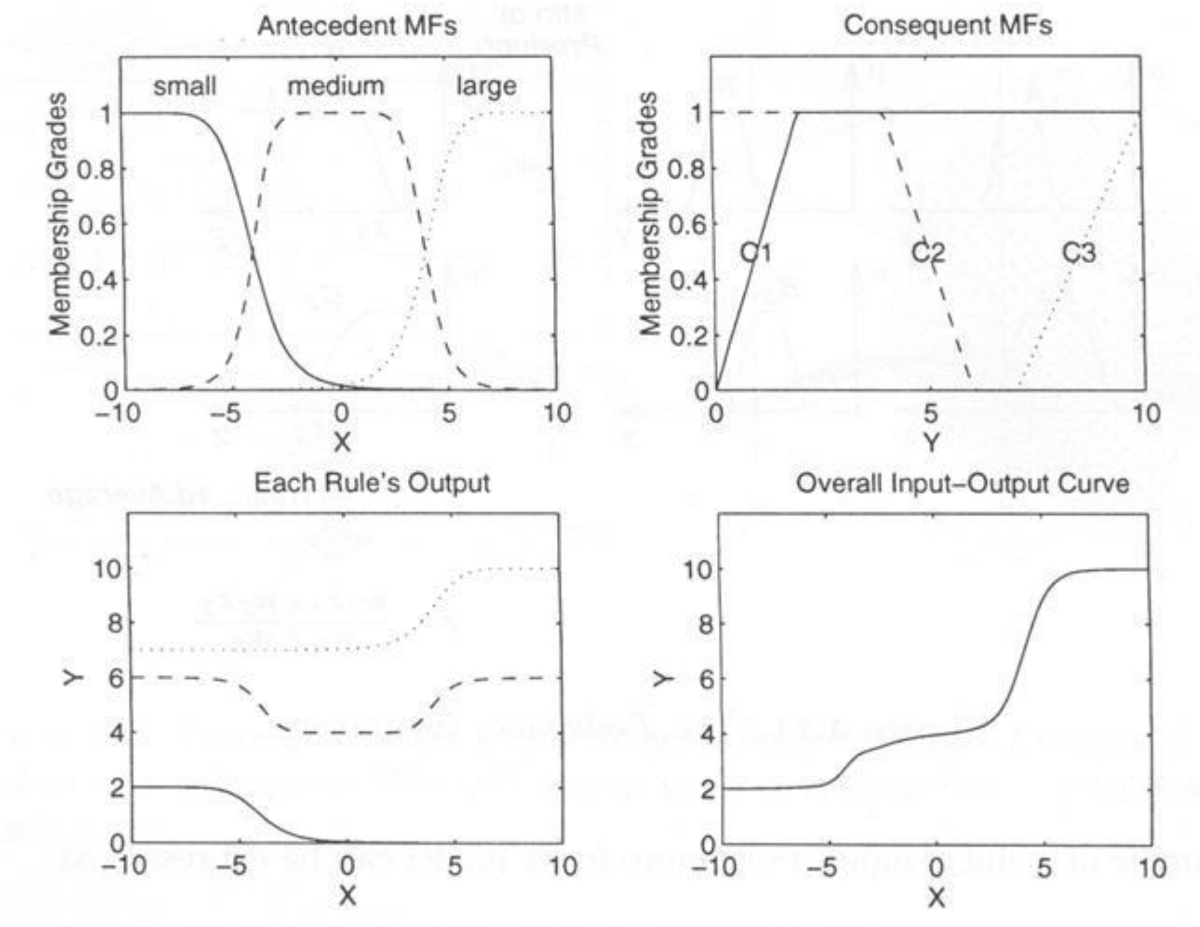


Fig. Output of Tsukamoto fuzzy model

Input Space Partitioning:

The antecedent of a fuzzy rule defines a local fuzzy region, while the consequent describes the behavior with in the region via various constituents. The consequent constituents are different MF or equation or constant depending on the fuzzy model. But, antecedents of fuzzy rules can be formed by partitioning the input space.

3 types-

Grid partition: Often chosen method. Applicable to small no. of input variables and MFs i.e. curse of dimensionality .

Tree partition: Each region is specified uniquely along a corresponding decision tree. Exponenential increase of no. of rules is reduced. More MFs are needed. Orthogonality holds roughly. Used in CART algorithm.

Scatter partition:

Portioning is scattered. Orthogonality doesn‛‟t hold. The portioned regions are non uniform.No. of rules is reduced, but overall mapping from consequent of each rule out put is difficult to estimate.

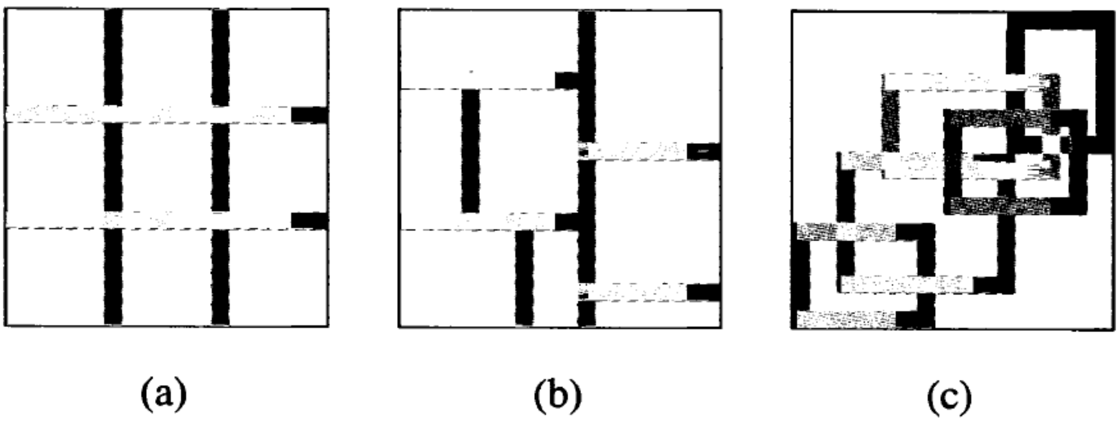


Fig. (a) Grid partition (b) Tree partition (c) Scatter partition

If certain transformation of the input is done, more flexible boundaries and partition will be obtained.

LECTURE-10

Defuzzification methods in detail:

strategy employed in Mamdani's fuzzy logic contr

Defuzzification

It refers to a way a crisp value is extracted from a fuzzy set as a representative value

5 methods of defuzzification

(i) Centroid of area z_{COA} (most widely used)

$$z_{COA} = \frac{\int_Z \mu_A(z)\, z\, dz}{\int_Z \mu_A(z)\, dz}$$

Where, $\mu_A(z)$ is the aggregated o/p MF.

(ii) Bisector of area z_{BOA}

z_{BOA} satisfies

$$\int_{\alpha}^{z_{BOA}} \mu_A(z).dz = \int_{z_{BOA}}^{\beta} \mu_A(z).dz$$

Where $\alpha = \min\{z \mid z \in Z\}$ $\beta = \max\{z \mid z \in$

i.e. z_{BOA} line partitions the region betn $z = \alpha$ $z = \beta$, $y = 0$ (y coordinate not var. y) & $y = \mu_A(z)$ into 2 regions with sam area.

(iii) Mean of maximum z_{MOM}

z_{MOM} is the average of the maximizing z at which the MF reach a maxn μ^*.

$$z_{MOM} = \frac{\int_{Z'} z\, dz}{\int_{Z'} dz}$$ where $z' = \{z \mid \mu_A(z) = \mu$

In particular, if $\mu_A(z)$ has a single max^m at $z = z^*$, then $z_{mom} = z^*$.

Moreover if $\mu_A(z)$ reaches its max^m whenever $z \in [z_{left}, z_{right}]$ then $z_{mom} = \left(\frac{z_{left} + z_{right}}{2}\right)$

(iv) Smallest of max.^{m} Z_{SOM}

Z_{SOM} is the minimum (in terms of magnitude) of the maximizing z.

(v) Largest of max^m Z_{LOM}

Z_{LOM} is the max.^{m} (in terms of magnitude) of the maximizing z.

Fuzzy Expert system

Expert systems are primarily built for the purpose of making the experience, understanding & problem solving capabilities of the expert in a particular subject area available to the non expert in this area.

The kernel of any expert system consists of

(i) A knowledge base (also called a longterm memory)

(ii) A database (or a short term memory or a blackboard interface)

(iii) An inference engine

(i) Knowledge base

Contains general knowledge pertaining to the problem domain.

In fuzzy expert systems, the knowledge is usually represented by a set of fuzzy production rules which connect antecedents with consequences, premises with conclusions or conditions with action.

Most common form

If A, then B, where A & B are fuzzy sets.

(ii) Data base

The purpose of the data base is to store data for each specific task of the expert system. Such as parameters of the problem or other relevant facts.

The data may be obtained from a dialog betn the expert system & the user. Other data may be obtained by the inference of the expert system.

(iii) Inference Engine

The inference engine of a fuzzy expert system operates on a series of ~~data~~ production rules and makes fuzzy inferences.

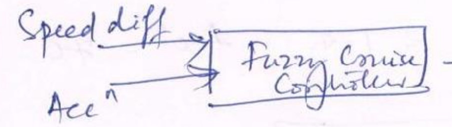
There exist 2 approaches to evaluating relevant production rules.

(i) Data driven – exemplified by the generalised modus ponens. In this case available data are supplied to the expert system, which then uses them to evaluate relevant production rules & draw all possible conclusions.

(ii) Goal driven. exemplified by the generalised modus tollens form of logical inference.

Gregnerot's Fuzzy Cruise Control.

To maintain a vehicle at desired speed



Fuzzy Rule base (As per Expert's experience)

Rule 1 If (speed diff. is NL) & acc^n ZE then throttle

Rule 2 ZE & NL then PL

Rule 3 NP & ZE then PM

Rule 4 NS & PS then PS

Rule 5 PS & NS then NS

Rule 6 PL & ZE then NL

Rule 7 ZE & NS then PS

Rule 8 ZE & NM then PM

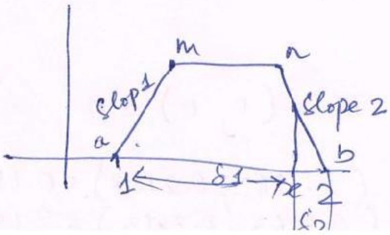
NL → Negative large
PM → Positive medium
NS → Neg.

S → Small
M → Medium
L

7 membership → NL NM NS ZE PS PM PL

For fuzzification of I/P & to compute the membership of antecedents

Slope known = 1



$\delta 1 = x - pt\ 1$

$\delta_2 = pt\ 2 - x$

If $\delta 1 \le 0$ or $\delta 2 \le 0$ $\mu = 0$

else $\mu = \min \begin{bmatrix} \delta 1 \cdot slope 1 \\ \delta 2 \cdot slope 2 \\ Max \end{bmatrix}$

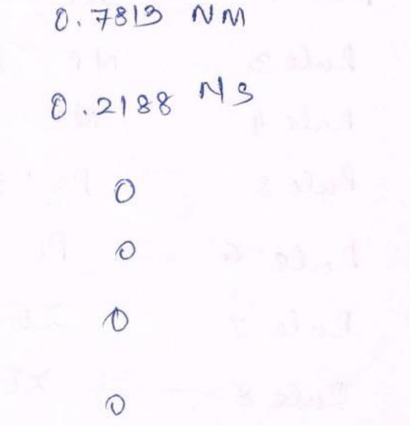
membership of x is Computed in all 7 member
μ_1', μ_2', μ_7' recorded in data str.

Eg Let. Speed diff = 100 Accⁿ = 70

I/P → Speed Diff → 100
I/P → Accⁿ → 70

Speed Diff (100)		Accⁿ (70)	
0	NL	0	NL
0	NM	0.7813	NM
0.8436	NS	0.2188	NS
0.1563	ZE	0	
0	PS	0	
0	PM	0	
0	PL	0	

For triangular



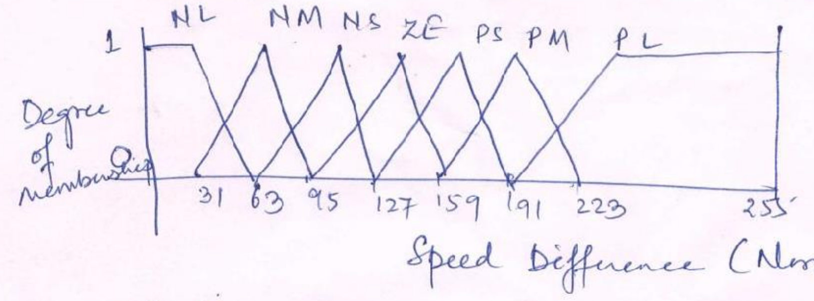
for NS $\delta 1 = 100 - 63 = 37$
$\delta_2 = 127 - 100 = 27$
$\text{Slope } 1 = 1/32 = 0.03125$
$\text{Slop } 2 = 1/32$

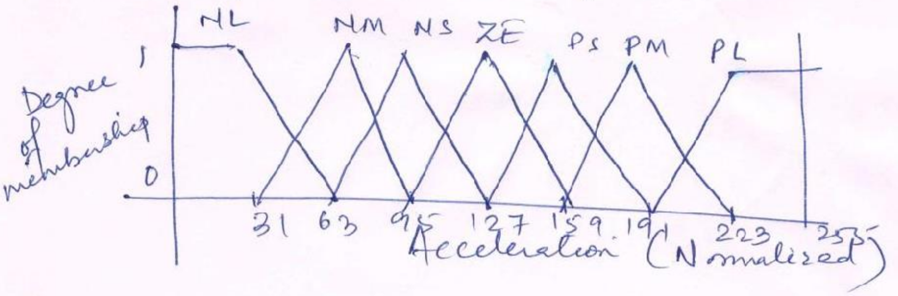
$$\mu_{NS}(x) = \min\begin{pmatrix} 37 \times 0.03125 \\ 37 \times 0.03125 \\ 1 \end{pmatrix} = 0.8438$$

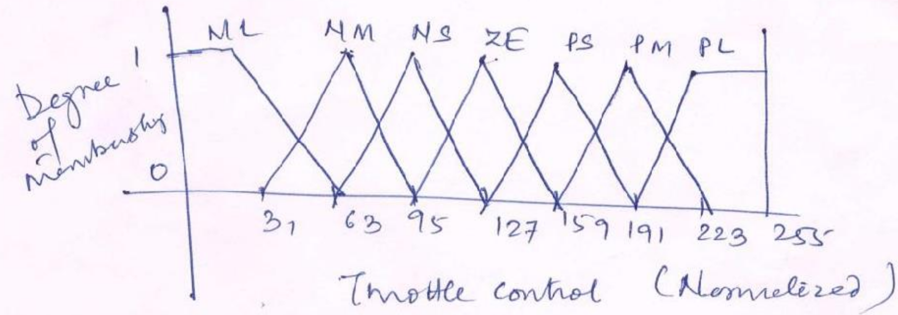
Rule strength compⁿ

Rule 1 min (ZE, NL) = min (0, 0) = 0
⋮
Rule 7 min (ZE, NS) = (0.1563, 0.2188) = 0.156
Rule 8 min (ZE, NM) = (0.1563, 0.7813) = 0.156

Greg's Throttle control
Fig of membership for
Continued.
Page ②







Q. What are the steps of fuzzy reasoning?

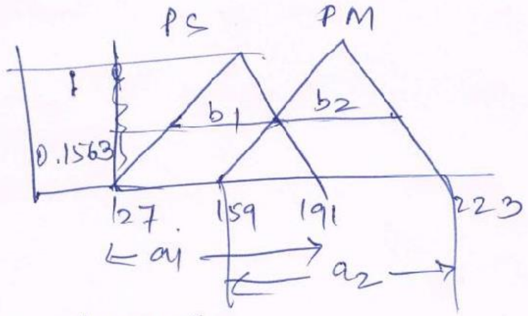
Q. What are the steps of fuzzy modelling?

Fuzzy O/P Or

As more than one rule qualify

Deffuzzi^n

O/P



~~Centroid~~ = $\frac{127+223}{2}$ =

for Ps centroid = 159

for PM = 191

Area = $\frac{1}{2} h(a_1+b_1)$

$= \frac{1}{2}(0.1563)(64+\ldots$

$= 9.99$

= 9.99

Weighted Avg CG = $\frac{9.99\times159 + 9.99\times191}{9.99+9.99}$

= 175 → throttle set